Compliance and Command I - Categorical Imperatives

The main aim of this series of papers is to develop a truthmaker semantics for the logic of imperative and deontic sentences. The first part deals with categorical imperative sentences, the second with deontic sentences and their interplay with categorical imperative sentences, and the third part with the interplay between indicative, imperative and deontic sentences and with conditional imperative and deontic sentences in particular. It would be helpful, though not strictly necessary, to have some standard exposition of truthmaker semantics at hand (such as Fine [2105]). I have for the most part been content with informal exposition but the reader may consult the appendix for some technical detail.

There are two central ideas behind the application of truthmaker approach to the semantics and logic of imperatives. The first is that the semantics is to be understood in terms of the actions which are in compliance with or in contravention to a given imperative sentence. However, in line with the general spirit of the truthmaker approach, compliance and contravention are taken to be *exact*. The compliant or contravening actions must be wholly relevant to the imperative with which it complies or to which it contravenes. Thus shutting the door is compliant with the imperative 'Shut the door!' while shutting the door and opening the window is not.

The second idea is that the validity of an imperative inference is to be understood in terms of the mereological relationship between the actions in compliance with the premiss and conclusion of the inference. More particularly, we should be able to see the content of the conclusion as part of the content of the premiss, where this is a matter of every action in compliance with the conclusion being part of an action in compliance with the premiss and every action in compliance with the premiss having as a part an action in compliance with the conclusion.

It is worth mentioning some key features of the account which set it apart from some other, prominent, accounts in the literature. In the first place, the account is hyperintensional. Indeed, certain truth-functionally equivalent sentences will not be assigned the same semantic or logical role. Thus even though p is truthfunctionally equivalent to $p \vee (p \wedge q)$, the account will distinguish between 'Raise a hand!' versus 'raise one or both hands', on the grounds that raising both hands complies with the second imperative though not the first; and the inference from either imperative to the other will not in fact be valid. I believe that the adoption of an intensional stance (and the adherence to the possible worlds framework, in particular) has been one of the main obstacles to developing a satisfactory semantics and logic for imperatives.

In the second place, the account is action- rather than outcome-oriented. The semantics for imperative is not given in terms of the *result* of complying with the imperative but in terms of the *action* by which compliance is achieved. In this regard, the account differs from the 'static' approach (as in Chellas [1971], for example), under which the content of an imperative is represented by a set of worlds, those which might result from its having been obeyed (or appropriately complied with); and it also differs from the 'dynamic' approach (as in Segerberg [1989], for example), under which the content of an imperative is represented by a 'transition' relation on worlds, relating a given state of the world to any of the states which might result from the imperative having been obeyed.

Of course, there is a way in which the dynamic approach is also action-oriented. For the transition relations correspond to the actions by which they are induced. But the actions themselves are represented in terms of the result of performing those actions, whereas we leave them alone, so to speak, and simply regard them as among the states or events in the world. It is possible to connect actions with transition relations on our approach, but they are not conceived as such and nor do these relations play any role in specifying the basic semantics.

In the third place, we achieve a kind of unity in the semantical and logical treatment of imperatives and indicatives. For the content of an indicative sentence is given by the states that verify it while the content of an imperative sentence is given by the actions that comply with it. Thus verification is to indicatives as compliance is to imperatives; and the semantical treatment of the two notions and the corresponding account of valid inference can be seen to run on parallel tracks.

This unity is lost on the dynamic approach since the content of an indicative sentence (a set of worlds) is essentially different from the content of an imperative (a relation on worlds); and so no uniform semantic treatment of the two cases is possible. Some unity can be achieved on the outcome-oriented approach (which no doubt has been regarded as a strong point in its favor) but without any of the benefits that accrue from giving the semantics of imperatives in terms of actions rather than worlds. Thus the unity is achieved on the present approach by thinking of a state as verifying an indicative in much the same way as an action complies with an imperative. From this point of view, the apparent inapplicability of a truth-conditional model of meaning to imperatives lies not in an inadequacy in the truth-conditional model as such but in an inadequate conception of what it is.

I should perhaps make a few remarks about my general approach to the topic. My interest is in what one might call the underlying semantics and logic for imperatives. The linguistic evidence in this area, as in any other, is incredibly complex. There are all kinds of factors at work; and if one is to make any progress in understanding the evidence at hand, then one must isolate a set of factors, which work together in explaining some significant aspect of it. In this regard, it is unfortunate that there is no language of the gods, in analogy with the celestial motion of the planets, that is relatively free from the kind of interference that prevents one from clearly discerning what general principles might be in play.

This means that I have not attempted to take every aspect of our actual use into account. I have been as much guided by a sense of what might be of logical significance in our usage than by a sensitivity to the actual form that it takes. It is perhaps worth recalling, in this connection, that the development of classical logic - one of the great intellectual achievements of the early twentieth century - was achieved neither by close attention to ordinary usage nor by ignoring it altogether but by attempting to isolate a significant aspect of that usage. And it is much the same spirit that has guided my own, more modest, efforts in the present paper.

Of course, the attempt to get at what I have called the underlying logic and semantics is not irrelevant to the understanding of natural language. It serves as a first step and as a useful foil against which other more realistic attempts may be evaluated. But it also serves a further important purpose, one that is to some extent at odds with the attempt to understand natural language. For it provides a basis for formalization, for systematically regimenting what we want to say and how we want to reason; and , in the present computational age, it also provides us with the benefits that come from implementing such a regime. My view is that the current 'linguistics turn' in the philosophy of language, while beneficial in many respects, has also been detrimental in deflecting attention away from some of the more traditional concerns of logical analysis.

The plan of the present part of the paper is as follows. I begin by outlining truthmaker semantics for indicative logic (§1); I show how the same form of semantics applies to imperatives (§2); I turn to the concept of validity for imperative inference, giving both a brief informal account (§3) and a formal account (§4); I then look at the logic of imperatives - considering the connection between various logical notions (§5), setting up a formal system of reasoning (§6), and discussing some inferences of special interest (§7); and I conclude with a comparison between my account and some related accounts in the literature (§8).

I myself developed the central ideas of the paper around 2010, having lectured on them in a number of venues and presented them in a number of seminars.[1] However, the reader should bear in mind that several other philosophers and linguists have developed very similar ideas over the last decade or so. Despite the similarities, my account differs in a number of significant respects from theirs: it is far more abstract and general and, what perhaps is of special importance, it is not in any way tied to a representation of states or actions as sets of worlds as or sets of atomic sentences and their negations or as sets of atomic states, as is so commonly assumed; it countenances a rich ontology of impossible actions, which play an important role in the development of the theory; it comes with a precisely formulated logic, something notably absent in many other accounts; it is unencumbered by inessential detail and is, both mathematically more manageable and philosophically more perspicuous in its formulation; and, as we shall later see, it better serves as a basis for extending the logic to other idioms, including the quantifiers and the conditional.

§1 Truthmaker Semantics for Indicative Sentential Logic

Let us briefly review truthmaker semantics for classical sentential logic (a fuller exposition can be found in Fine [2015]).

Indicative formulas are constructed in the usual way from indicative sentential atoms p_1, p_2, ... by means of negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$) and the verum constant $\top$.

We take each sentence to be *verified* or *falsified* by a state. It is integral to our understanding of verification and falsification that they be exact, i.e. that the verifier or falsifier should be wholly relevant to the sentence that is verifies or falsifies. Thus a state which contains an exact verifier as a proper part will not itself, as a rule, be an exact verifier.

More formally, we suppose given a *state space*, which is a set of candidate truthmakers and falsemakers, called 'states', ordered by part-whole. We may also draw a distinction between possible and impossible states (with every part of a possible state being a possible state); and in

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[1]I should like to thank the participants at these lectures and seminars for many helpful comments. I might add, on a more personal note, that I first became interested in imperative inference as an undergraduate around 1966, when I read the papers of Kenny [1966] and Geach [1966]. I came up with the idea of truth maker semantics around 1969 and had even considered the application to imperative logic, but it took me almost half a century to appreciate the full scope and significance of the approach.

that case, we may say that two states are *compatible* when their fusion is a possible state and that otherwise they are *incompatible*.

We assume that, in a state space, the fusion of any number of states is also a state. This means, in particular, that there will exist the fusion of no states, the *null* state □, and the fusion of all states, the *full* state ■. The null state □ will be a part of every state and nothing is required for it to obtain; and the full state ■ will contain every state and everything is required for it to obtain.

Normally, a state space will contain impossible states since the fusion of two possible but incompatible states will be an impossible state; and so, in particular, ■ will be an impossible state. However, a state space may contain many different impossible states besides ■, the most impossible of the impossible states. The state of an object's being red and green, for example, might be distinguished from the state of its being round and square or from the state of its being round and square and red in terms of its composition from the component possible states.[2] In the special case in which ■ is the sole impossible state, we shall say that the state space is *topsy*; and, similarly, in the special case in which □ is the sole necessary state, one compatible with every possible state, we shall say that the state space is *topsy*. And, of course, a space will be *topsy-turvy* when it is both topsy and turvy, with a single impossible state on top and a single necessary state at the bottom.

A *state space model* is a state space accompanied by an assignment of a set of verifiers and a set of falsifiers to each indicative atom. We assume, in general, that the two sets are non-empty, i.e. that each atom has both a verifier and a falsifier. But we should note that the semantic values of atoms are in no way special; the content of any formula, no matter how complex, is capable of serving as the semantic value of an atom. This is a desirable feature of a semantics and often lacking in the truthmaker-style of semantics to be found in the literature.

We have the following clauses for when a conjunction or a disjunction or verum is verified by a state within a model:

(1) the state s verifies the conjunction T $\wedge$ U iff it is the fusion $t \sqcup u$ of states t and u which respectively verify its conjuncts T and U;

(2) s verifies the disjunction T $\vee$ U iff s it verifies one of its disjuncts, T or U;

(3) s verifies $\top$ iff s is identical to the null state □.

For negation, we make use of the notion of (exact) falsification:

(4) s verifies a negation ¬S iff it falsifies the negated indicative S.

We then introduce 'dual' clauses for when a conjunctive or disjunctive sentence is falsified:

(1′) s falsifies the conjunction T $\wedge$ U iff s falsifies one of its conjuncts, T or U;

(2′) s falsifies the disjunction T $\vee$ U iff it is the fusion $t \sqcup u$ of states t and u which respectively falsify its disjuncts T and U;

(3′) s falsifies $\top$ iff s is identical to the full state ■;

(4′) s falsifies the negation ¬S iff it verifies the negated indicative S.

We might define $\perp$ as $\neg\top$. The clauses for $\perp$ are then the reverse of the clauses for $\top$:

(5) s verifies $\perp$ iff s is identical to the full state ■;

[2] Fine [2016c] shows how a multitude of impossible states can be naturally introduced into a state space containing only possible states.

(5)′ *s* falsifies $\perp$ iff *s* is identical to the null state $\square$.

Within the present framework, we might identify a (*propositional*) *content* or *proposition* either with a set of states (the *unilateral* content) or with a pair of sets of states (the *bilateral* content) and we might take the (*positive*) *content* $[S]^+$ *of* a sentence S to be the set *V* of its verifiers, its *negative content* $[S]^-$ to be the set *F* of its falsifiers, and its *bilateral* or *full content* [S] to be the pairing $<V, F>$ of its positive and negative content. It is readily shown that every formula, like every atom, will have both a verifier and a falsifier, although the verifier - as with p $\wedge \neg$p - may be an impossible state.

We may introduce two notions of entailment - the familiar notion of classical entailment and the less familiar notion of analytic entailment. Say that a model is *classical* if (i) any verifier of an indicative atom is incompatible with any of its falsifiers and if (ii) for any atom, any possible state is compatible with a verifier of the atom or with a falsifier of the atom. Sentence S then *classically entails* sentence T if no verifier for S in any classical model is compatible with a falsifier for T. On the other hand, S will *analytically entail* T if (i) every verifier for S contains a verifier for T and (ii) every verifier for T is contained in a verifier for S.[3] Similar relations, here and elsewhere, can be defined to hold between the contents themselves, rather than between the sentences with those contents.

Classical entailment corresponds, as one might expect, to the notion of classical tautological implication, while analytic entailment corresponds to a slight variant of Angell's notion of analytic implication.[4] It might plausibly be argued that the notion of analytic entailment captures the intuitive idea of the content of one sentence being *part* of the content of another (Fine [2015], Yablo [2014]).

§2 Truthmaker Semantics for Imperative Sentential Logic

We turn to the corresponding form of truthmaker semantics for imperatives. We may suppose that imperative formulas are constructed in the usual way from imperative sentential atoms α_1, α_2, ... by means of negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$) and the verum constant $\top$.

We might think of the atoms as corresponding to simple imperative sentences such as 'shut the door', 'turn on the light' or 'open the window'. We are then able to form negations, such as 'do not shut the door' or 'do not open the window', conjunctions, such as 'shut the door and open the window', and disjunctions, such as 'shut the door or open the window'. More complicated constructions are also possible, of course, as with 'do not both shut the door and open the window' or 'shut the door and either open the window or turn on the light'.

In the case of indicatives, we have taken the basic semantic notions to be ones in which a state verifies or falsifies a sentence. Similarly, in the case of imperative sentences, we take the

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[3]We might also add the condition that every falsifier for T should be contained in a falsifier for S. This results in a somewhat different notion of entailment. For p $\wedge$ (q $\vee$ r) will entail (p $\wedge$ q) $\vee$ (p $\vee$ r) without the addition, though not with the addition.

[4]To be exact, it is the first variant system of §11 of Fine [2015].

basic semantic notions to be ones in which an action is in *compliance with* or is in *contravention of* an imperative. We might also talk in this connection of the imperative *allowing* or *disallowing* the action. Thus the action of shutting the door will be in compliance with the imperative 'shut the door', while the action of leaving it open will be in contravention of the action.

We might well regard an action as a special case of a state, i.e. as being a candidate verifier or falsifier though, for certain purposes, we might wish to distinguish an action from the performance of the action, which would then be the state proper. Actions might be construed as action types not tied to any particular agent (shutting the door), or action types tied to a particular agent (your shutting the door), or action tokens (your shutting the door on such and such an occasion). How we take them will not much matter for present purposes, although we should always suppose that our imperatives relate to a fixed agent or fixed group of agents, to a fixed moment of time, and perhaps also to a fixed decision-theoretic situation.

We regard an action space as a special case of a state space, one in which all the states are actions; and we suppose that it is subject to the standard conditions on a state space. In particular, actions will be closed under arbitrary fusion, so that any sequence or combination of actions, no matter how large or gerrymandered, will also constitute an action.

Just as with verification or falsification, compliance or contravention is understood to be *exact*; the action must be in compliance or contravention with the imperative as a whole. So whereas the action of shutting the door will be in compliance with the imperative 'shut the door', the action of shutting the door and opening the window will not, in the relevant sense, be in compliance with the imperative; and similarly, whereas the action of leaving the door open will be in contravention of the imperative 'shut the door', the action of leaving the door open and closing the window will not, in the relevant sense, be in contravention of the imperative.

There is perhaps but one action, shutting the door, that is in compliance with the imperative 'shut the door'; and also, though somewhat more problematically, there is perhaps but one action, leaving the door open, that is in contravention of the imperative. But, in general, there may be more than one action in compliance with or in contravention of a given imperative. Thus two actions, shutting the door or opening the window, will comply with the imperative 'shut the door or open the window', and two actions, leaving the door open or the window closed, will be in contravention of the imperative 'shut the door and open the window'. This point is of some importance, for often, under an action-oriented semantics for imperatives, it is supposed that there is but one action in compliance with an imperative - as given perhaps by some transition relation on worlds - and no meaningful sense in which we are provided with a *choice* of actions.

Among the actions to be considered will be the null action, the fusion of zero actions, which we also designate as $\square$. It is an action that will be part of any other action and which one does (as part of) whatever one does. Among the actions may also be impossible actions, such as shutting the door and leaving it open, which results from the fusing of possible actions.

When it comes to the compliance and contravention conditions for arbitrary imperative formulas, we have exact analogues of the clauses for indicatives:

(1) the action a complies with the conjunction $X \wedge Y$ iff it is the fusion $b \sqcup c$ of actions b and c which respectively comply with its conjuncts X and Y;

(2) a complies with the disjunction X $\vee$ Y iff it complies with one of its disjuncts, X or Y;

(3) a complies with $\top$ iff a is identical to the null action $\square$;

(4) a complies with the negation $\neg$X iff it contravenes the negated imperative X;

(1$'$) a contravenes a conjunction X $\wedge$ Y iff it contravenes one of its conjuncts, X or Y;

(2$'$) a contravenes the disjunction X $\vee$ Y iff it is the fusion $b \sqcup c$ of actions b and c which respectively contravene its disjuncts X and Y;

(3$'$) a contravenes $\top$ iff a is identical to the full action $\blacksquare$;

(4$'$) a contravenes the negation $\neg$X iff it complies with the negated imperative X.

Of special interest is clause (3) for compliance with the null imperative $\top$; the one and only way to comply with $\top$ is to perform the null action $\square$. Accordingly, we might read $\top$ as 'do nothing'. But this has to be appropriately understood. It does not mean 'perform no action at all', nor 'do nothing in particular, i.e. anything you like', but rather 'perform that particular action which requires nothing of you'. Thus the nothing here is not a nothing at all, nor an anything whatever, but a something that amounts to nothing (my poor attempt to outdo Heidegger); and compliance with the imperative $\top$ merely requires null compliance, i.e. the performance of the null action $\square$.

As we have seen, three kinds of content may be associated with an indicative sentence - the (*positive*) *content*, the set of its verifiers, the *negative content*, the set of its falsifiers, and the *full content*, the positive and negative content taken together; and we might identify a *propositional content* or *proposition* with a set of states (its verifiers). Likewise, three kinds contents may be associated with an imperative sentence - the (positive) content, the set of actions it allows, a negative content, the set of actions it disallows, and the full content, the two taken together; and we might identify a *prescription* or *prescriptive content* with a set of actions (those that it allows).

There is clearly a close parallelism in our semantic treatment of indicative and imperative sentences. There is, first of all, a parallelism in our choice of semantical primitives, with verification corresponding to compliance and falsification to contravention. There is, in the second place, a parallelism in the semantical clauses, with the clauses for the one being the exact counterpart of the clauses for the other. Thus when S is an indicative sentence, such as 'you shut the door and open the window' and X = S! is the corresponding imperative, 'you, shut the door and open the window', then an action/state will verify or falsify S just in case it is in compliance with or in contravention to X.

There is a question as to what accounts for this parallelism, both at the syntactic and at the semantic level.[5] On one view, the syntax and semantics for imperatives *derives* from the syntax and semantics for indicatives (as suggested by McGinn [1977], for example). Thus any imperative X is syntactically derived from a corresponding indicative sentence S and the semantics for X is derived from the semantics for S by transforming its verification and falsification conditions into compliance and contravention conditions.

Another view, loosely deriving from Frege [1879], is that there is a common element to

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[5]There is an extended discussion of this question in chapter 1 of Mastop [2005]

an indicative S and the corresponding imperative S!, what Hare [1952] calls the 'phrastic'. These can then be taken to be subject to positive and negative 'realization conditions', which are the neutral counterpart of the verification and falsification conditions for indicatives and of the compliance and contravention conditions for imperatives. The syntax and semantics for indicatives and imperatives alike is then derived - by means of appropriate mood-indicators or 'neustics' in Hare's terminology - from the syntax and semantics for the phrastics, with realization being given a 'world-to-word' spin in the case of indicatives and a 'word-to-world' spin in the case of imperatives.

There are perhaps other ways of accounting for the parallelism. Fortunately, there will be no need for us to take a stand on this question or even on whether it is meaningful; and although we shall later appeal to the correspondence between indicatives and imperatives, it is without regard to which, if either, comes first.

Finally, let us note that the use of $\top$ enables us to make a remarkable simplification in our account of imperatives. For as many writers have observed, imperatives can come both with an obligatory and with a permissive force, as when one says 'take a cake' (if the commanding officer says 'take a cake', the cadet can appropriately say 'no, thanks', but not if the officer says 'salute!'). Despite the difference, we may reduce the one to the other by equating the permissive use of the imperative X with $X^{\top} = X \vee \top$, since the actions in compliance with $X \vee \top$ will be the actions in compliance with X or the null action $\Box$. Thus an alternative to complying with X will be to do nothing at all. The reduction might also help explain how the simple imperative is capable of being used with a permissive meaning.

§3 The Intuitive Concept of Imperative Validity

There would appear to be an intuitive sense in which one imperative may follow from others or from a combination of imperatives and indicatives. Imagine that A is an agent, C an authority who issues commands to the agent, and B an intermediary who attempts to interpret what C says for the benefit of A.[6] Suppose that A says 'it is raining' and 'if it is raining then take an umbrella'. Then it is perfectly appropriate for B to say to C, 'so take an umbrella'. Similarly, if A says 'take an umbrella' and 'put on a rain coat', then it is perfectly appropriate for B to say to A, 'so take an umbrella and put on a rain coat'.

In both cases, B appears, on the face of it, to make an inferential use of the word 'so', one which indicates that the ensuing sentence (the conclusion) does indeed follow from the preceding sentences (the premisses). This suggests that B has performed two inferences; and the fact that it is appropriate for B to say what he does suggests that the inferences are valid, that the imperative conclusion in each case does indeed follow from its premisses. But whether or not there is a genuine inference here, it seems clear that the transition from the one imperative sentence to the other is made on the basis of a logical relation between the two sentences; and so we would like to know what that relation is and which other sentences also stand in the relation. In this way, we can avoid the contentious question of whether the relation is genuine inferential.

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[6]B could, in principle, be identical to C but this raises other issues, which I prefer to put on one side.

There are some related logical notions which also appear to make good intuitive sense in application to imperatives. Suppose C says 'shut the door or open the window'. Then an equivalent imperative is 'open the window or shut the door'; and nothing would be lost if, in reporting what C had told A to do, B had said the one rather than the other. Or again, suppose C says 'shut the door' and 'leave the door open'. Then in a clear sense, the two imperatives are inconsistent; the one is, as a matter of logic, in 'conflict' with the other.

Moreover, the application of these various logical notions to imperatives is of general importance. For instructions or directives are part of the warp and woof of everyday life - whether they issue from ourselves, or from those who have authority over us, or from the institutions or governing bodies to which we belong. For to understand what to do in the face of some instructions is often a matter of knowing what follows from them, to know whether we have correctly interpreted some instructions is a matter of knowing whether one formulation of them is equivalent to another, and to know whether we can even do what we have been instructed to do is, in part, to know whether the instructions are consistent.

These logical notions also apply to indicative sentences, of course. But we should wary of assuming that their application to imperatives is a straightforward analogue of their application to indicatives; and we should be wary, in particular, of assuming that the validity of an imperative inference will consist in the preservation of some straightforward imperative analogue of truth or that the logic of such inferences will be a straightforward analogue of classical logic.

In this regard, the 'paradox' of Ross [1941] assumes great importance (and similarly for 'free choice' effects). For the main reason, it seems to me, for accepting the inference from 'shut the door' to 'shut the door or burn the house down' is the validity of the corresponding inference in classical logic. But if we no longer take the correspondence with classical logic for granted, then we are under an obligation to find an account of validity for imperative inference in which such inferences will no longer be valid and in which the correspondence with classical validity can no longer be sustained.

Of course, it would be desirable to arrive at a common conception of validity for indicative and imperative inference and a common set of inference rules. This is not ruled out. But such commonality, if it is to be had, will lie in finding an alternative approach to indicative logic rather than in extending the classical approach to imperative logic.

§4 Validity Defined

I wish now to propose an account of imperative validity which I believe is largely in conformity with our intuitions and which also makes good sense of the notion. It is evident from the definition why the notion is as it is.

The proposal is that the imperative X will *entail* the imperative Y - or, equivalently, that the inference from X to Y will be *valid* - just in case (in any model) (i) any action in compliance with X contains an action in compliance with Y and (ii) every action in compliance with Y is contained in an action in compliance with X. When the inference is from several imperatives X_1, X_2, ... to a given imperative Y, the inference is taken to be valid just in case the inference from the conjunction $X_1 \wedge X_2 \wedge ...$ to Y is valid in the previous sense, i.e. just in case (i) any fusion of actions in compliance with each of X_1, X_2, ... contains an action in compliance with Y and (ii) any action in compliance with Y is contained in the fusion of actions in compliance with X_1, X_2,

... .[7]

Thus the notion of entailment between imperatives is the exact analogue of the notion of analytic entailment between indicatives; and, indeed, we might with some justice claim that for X to entail Y is for the content of Y to be *part* of the content of X. Entailment, in the case of imperatives, is the articulation of prescriptive content.

The above account involves two requirements: (i) that X *subsumes* Y; and (ii) that Y *subserves* X. Subsumption and subservience can, in their turn, be explained in terms of of partial and entire compliance. An action *a partially complies with* an imperative X if it is contained in an action that exactly complies with X and the action *a entirely complies with* X if it contains an action that exactly complies with X. Then X subsumes Y iff:

any action that entirely complies with X entirely complies with Y

or, equivalently, iff:

any action that exactly complies with X entirely complies with Y,

and Y subserves Y iff:

any action that partially complies with Y partially complies with X

or, equivalently, iff:

any action that exactly complies with Y partially complies with X.

Thus the validity of the imperative inference X/Y amounts to the preservation of entire compliance from left to right and the preservation of partial compliance from right to left. Or to pur it more intuitively, the imperative inference X/Y will be valid just in case:

(i) any action in compliance with the premiss must go all of the way towards complying with the conclusion; and

(ii) any action in compliance with the conclusion must go some of the way towards complying with the premiss.

We might think of the validity of the imperative inference in terms of means-end reasoning. A means here is a *partial constitutive* means, i.e. one that goes some way to constituting the end. The validity of an imperative inference then consists in the conclusion being a necessary means to the conclusion. The conclusion is a (constitutive) *means* since any action in compliance with the conclusion is contained in an action in compliance with the premiss (the subservience requirement); and it is a (constitutively) *necessary* means since any action in compliance with the premiss guarantees, via some part, compliance with the conclusion (the subsumption requirement).

We might also think in terms of an imperative as having a double role or meaning. Suppose that a_1, a_2, ... are the actions in compliance with X. Then we might regard X as making obligatory one of the actions a_1, a_2, ... and as making permissible each of the actions a_1, a_2, ... Thus the imperative conveys in this way both an obligation and a permission.[8]

[7] As before, we might add a further clause to the effect that every action in contravention of Y should be contained in an action in contravention of X, although I am inclined to think that our focus in imperative inference is on compliance rather than contravention.

[8] I believe that it may also be useful to think, in a similar way, of an indicative sentence as requiring one of its verifiers to obtain and of allowing each of them to be obtain.

We might now think of the validity of the imperative inference X/Y in terms of the preservation of this double aspect: we need some guarantee that whatever is made obligatory or permissible by X is also made obligatory or permissible by Y. But what does such a guarantee amount to? Suppose that the obligation made by X is satisfied, i.e. that one of the actions in compliance with X is performed. Then what serves to guarantee that one of the actions in compliance with Y is obligatory is that the action *a* in compliance with X, whatever it might be, should contain an action *b* in compliance with Y - which is Subsumption. Suppose now that each of the actions in compliance with X is permitted. Then what serves to guarantee that each action in compliance with Y is permitted is that each such action *b* should be part of an action *a* permitted by X - which is Subservience. (This understanding of imperatives will later be useful in working out their connection with deontic statements and in developing a preservationist conception of validity).

The above account of validity for imperative inference goes some way towards achieving the desired unity with indicative inference. For there is a notion of partial content for indicative sentences, of independent standing, under which the indicative inference S/T is only taken to be valid when the content of T is *part* of the content of S. But this then is the very same as the criterion of validity for imperative inference; the inference X/Y will be valid just in case the content of the conclusion Y is part of the content of the premiss X.

However, a disparity remains. For the notion of classical entailment, essentially the preservation of truth, is of great, perhaps pre-eminent, significance to the logic of indicatives. What corresponds to truth for indicatives is satisfaction for imperatives, where an imperative is satisfied in given circumstances if it is obeyed (or suitably complied with) in those circumstances. However, the corresponding notion of validity for imperatives is of relatively little, if any, significance.

What accounts for the disparity, I believe, is a difference in our concerns. We have a great interest in the preservation of truth, in which sentences will be true given that other sentences are true. Thus this is a principal way in which we may extend our knowledge or discern errors in what we presume to know. However, we have little or no interest in the preservation of satisfaction as such. Given that I obey certain imperatives, then why should I have any interest in which other imperatives are thereby obeyed? My interest is in compliance rather than satisfaction. Given certain imperatives, my interest is in *how* I might comply with them and I normally only have an interest in which other imperatives are thereby satisfied in so far as their satisfaction provides a guide to compliance. The notion of validity of interest to us is therefore one that reflects this concern.

Thus I do not think it is altogether correct to say that there is one notion of validity - or, better, entailment - for indicative inference and another for imperative notion. There are two notions in each case, but one is of pre-eminent (though not sole) significance for indicatives and the other of pre-eminent (and perhaps even sole) significance for imperatives.

§5 Other Logical Notions

We discuss the sense in which some imperatives might be unsatisfiable (or inconsistent) or in which one imperative might be equivalent to another; and we then relate these two notions to the previous notion of validity. We shall see that the logical landscape looks rather different from the usual classical perspective.

There are two senses in which a set of imperatives X_1, X_2, ... might be said to be inconsistent or unsatisfiable. They will be *strongly unsatisfiable* (in a classical model) if no possible action is in compliance with their conjunction and they will be *weakly unsatisfiable* if some impossible action is in compliance with their conjunction. Similarly, the imperatives X_1, X_2, ... will be *strongly satisfiable* if every action in compliance with their conjunction is possible and *weakly satisfiable* if some action is compliance with their conjunction is possible. With weak unsatisfiability, there is a kind of partial inconsistency while, with strong unsatifiability, there is a through-going inconsistency.[9]

So, for example, the imperative X = 'shut the door and do not shut the door' (of the form $\alpha \wedge \neg\alpha$) will be strongly unsatisfiable, since any action in compliance with X will be the fusion of an action *a* in compliance with the imperative and an incompatible action *a′* in contravention of X. On the other hand, the imperative X = 'either shut the door and do not shut the door or open the window' (of the form $(\alpha \wedge \neg\alpha) \vee \beta$) will be weakly unsatisfiable though not strongly unsatisfiable, since an impossible action in compliance with the first disjunct will be in compliance with the disjunction while a (presumably possible action) in compliance with the second disjunct will also be in compliance with the disjunction.

We might see the strong notion of satisfiability as arising from the obligatory aspect of imperatives. If the obligation to do one of the actions in conformity with the imperative is to be met, then one of those actions must be possible. Similarly, we might see the weak notion of satisfiability as arising from the permissive aspect of imperatives. If the permission to do each of the actions in compliance with the imperative is to be effective, then each of the actions must be possible.

The distinction is relevant to a puzzle discussed by Hare [1967] and Williams [1963] (and also by Veltman [2009]). For there seems to be some kind of contradiction involved in the imperative 'Help your mother or help your father, but don't help your mother'. But it is not the same kind of straight flat out contradiction that is involved in 'Help your mother and don't help your mother', since it is possible to comply with the first imperative (by helping your father) but not with the second.

The difference, for us, is simply the difference in the two kinds of unsatisfiability, with there being a satisfiable permission in the case of the second imperative though not the first and and an unsatisfiable permission in the case of both imperatives. But we should note that it is only by allowing impossible actions to be in compliance with an imperative that we are able to draw the distinction between the two kinds of contradiction. A more standard intensional approach, in which actions are taken to be non-empty sets of worlds, is not able to explain the

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[9]These notions of satisfiability and unsatisfiability are relative to a model but we might lift the relativity by taking satisfiability or unsatisfiability simpliciter, whether weak or strong, to be satisfiability or unsatisfiability in every model (or every classical model).

We might also take the imperatives X_1, X_2, ... to be be unsatisfiable if there is no action whatever in compliance with their conjunction $X_1 \wedge X_2 \wedge ...$, although this notion of unsatisfiability will only have application if we allow there to be imperatives which allow no actions.

difference in this way or, perhaps, in any way at all.

We turn to equivalence. We might say that the imperatives X and Y are *exactly equivalent in* a model if the same actions are in compliance with both within the model and that they are *exactly logically equivalent* if they are exactly equivalent in each model (a somewhat stronger definition would also require the same actions to be in contravention of X and Y).

We can provide an account of exact entailment in terms of exact equivalence, where X exactly entails Y if every action in (exact) compliance with X is in (exact) compliance with Y. For X will exactly entail Y just in case Y is exactly equivalent to X $\vee$ Y.[10] We can also provide a definition of strong inconsistency in terms of exact equivalence. For let $*$ be the thoroughly inconsistent imperative which allows all and only impossible actions. Then the imperative X will be strongly inconsistent just in case $*$ is exactly equivalent to X $\vee$ $*$.

There is an alternative account of equivalence as two-way analytic entailment. The imperatives X and Y will be equivalent in this sense just in case every action in compliance with X contains and is contained in an action in compliance with Y and if every action in compliance with Y contains and is contained in an action in compliance with X.

Perhaps somewhat surprisingly, the two definitions are not coextensive. For consider the imperatives X = $\alpha \vee (\alpha \wedge \beta \wedge \gamma)$ and Y = $\alpha \vee (\alpha \wedge \beta) \vee (\alpha \wedge \beta \wedge \gamma)$ - as in 'Press button A or buttons A, B and C' and 'Press button A or buttons A and B or buttons A, B and C'. Then even though each analytically entails the other, according to our definition, they do not allow the same actions. For the action of pressing buttons A and B will be in exact compliance with the second but not the first.

Some other standard connections between the different logical notions will also not hold. For example, the imperatives X = α and Y = $\beta \wedge \neg\beta$ will be unsatisfiable (in either the strong or weak sense). However, X does not entail $\neg$Y since an action in compliance with or contravention to β and hence in compliance with $\neg$Y need not be contained in an action in compliance with X.

Or again, it does not follow from the fact that X entails Y that X is equivalent (in even the weak sense) to X $\wedge$ Y. For X entails X. However, Idempotence fails, X is not in general equivalent to X $\wedge$ X. For suppose X allows both *b* and *c*. Then their fusion will be in compliance with X $\wedge$ X but not, in general, with X itself. To take an actual example 'Shut the door or open the window, and shut the door or open the window' will allow one to both shut the door and open the window, whereas plain 'Shut the door or open the window' will not. But this means that X will entail X even though X will not in general be equivalent to X $\wedge$ X.

The relationship between entailment and equivalence is somewhat more complicated. Suppose we are given two imperatives X and Y whose respective contents are $X = \{a_1, a_2, ...\}$ and $Y = \{b_1\ b_2, ...\}$. Then we may say that the imperative Z is a *coordinated conjunction* of X and Y if there is a relation R whose domain is X and whose range is Y and which is such that the content Z of Z is the set of all those fusions $a \sqcup b$ for which a stands in the relation R to b. Thus coordinated conjunction only requires pairing every member of X with a member of Y and every member of Y with a member of X, whereas ordinary conjunction requires that every member of X

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[10]In this respect, there is a difference between exact entailment and analytic entailment since, as we shall see, there is not an analogous definition of analytic entailment in terms of exact equivalence and conjunction.

be paired with every member of *Y* (the notion of coordinated conjunction is further discussed in Fine [2016b]).

It is now easy to show that X will entail Y just in case X is a *coordinated* conjunction of Y and X - or of Y and any other imperative Z, for that matter. We might think of the additional conjunct Z as telling us how to go on once we have complied with Y, where how we go on is a function of how we began (with a particular member of *Z* being paired with particular members of *Y*). Thus X will entail Y if there is a way to go on, given compliance with Y, that will then constitute compliance with X.

The failure of Idempotence is also relevant to the attempt to account for entailment in terms of updating. One familiar way to do this is as follows (Veltman [1996], p. 224; Mastop [2005]). We are given a context c, which may be updated by a sentence X, to give an updated context c[X]. X is then taken to entail Y just in case c[X][Y] = c[X] for any context c; updating with Y, after updating with X, gives nothing new. Within the present setting, we may naturally take a context c to be given by a (unilateral) content *C*. Updating is then a form of conjunction: where *X* is the unilateral content of X, c[X] = $C \wedge X$. But letting *C* be the 'null' context $\{\square\}$, c[X] = *X* and c[X][X] = $X \wedge X$ and, since *X* is not in general the same as $X \wedge X$, the account will not deliver the result that X entails X. To take an actual example, if I update with 'take an apple or a pear' and then update again with 'take an apple or a pair', then I am offered a choice which I did not have before, which is to take both an apple and a bear.

It might be thought that the problem could be overcome by adding the stipulation that A entails A. However, it can be shown that, according to the proposed criterion, no formula (free of $\top$) *ever* entails another formula; and so no simple patch of this sort can be made to work.

We could avoid these various anomalies by closing the content of an imperative under two conditions: Fusion, according to which the fusion of any non-empty set of actions in compliance with an imperative is also in compliance with the imperative; and Convexity, according to which any action which lies between two actions in compliance with an imperative is also in compliance with the imperative. There are contexts in which these conditions (or their alethic counterparts) can reasonably be imposed. If our interest is in partial truth or verisimilitude of a theory, for example, then nothing is lost by supposing that its content is closed under both of these conditions.

However, in the present context, Fusion is unacceptable. We do not want the imperative 'press button A or press button B', for example, to allow one to press both buttons, as would be the case with closure under Fusion. Moreover, if we start off with an inclusive concept of disjunction, under which $\alpha \vee \beta$ is equivalent to $\alpha \vee \beta \vee (\alpha \wedge \beta)$, then it will not be possible to define a non-inclusive concept, under which the equivalence fails, in terms of it. But we may, of course, define the inclusive notion in terms of the non-inclusive notion as $\alpha \vee \beta \vee (\alpha \wedge \beta)$. Thus considerations of expressivity require that we adopt a non-inclusive treatment of disjunction (and of other such connectives).

Convexity is perhaps also unacceptable, although the case is less clear. For the imperative 'press button A or press buttons A, B and C' does allow one, in a sense, to press buttons A and B, even though the pressing of A and B is not in *exact* compliance with the imperative. Thus if we close the content of an imperative under convexity, it will no longer be clear which actions are in exact compliance with the imperative although it will still be clear which actions are in partial compliance and also which actions are in partial compliance while still containing an action in full compliance with the imperative (and hence go 'far enough' without going 'too far').

It might be thought that the failure of strict equivalence to be coincide with two-way entailment points to a shortcoming in our notion of entailment. For our interest in drawing imperative consequences from some imperative premiss is not only in determining the necessary constitutive means by which we might comply with the premiss but also in determining how we might be in exact compliance with the premiss. It might be thought that this further requirement simply amounts to the conclusion entailing the premiss. But, as is shown by the button example, this is not so; and so we seem to lack the means of expressing this further requirement.

However, the situation is not so straightforward. For we need to distinguish two cases. The first is one in which the imperative premiss or premisses do not state all of the imperatives in force. In this case, the requirement that any action in compliance with the conclusion should be in compliance with the premisses is of no interest to us, since it may still not be in compliance with the other imperatives that are in force. The second case is one which the imperative premisses do state all the imperatives in force (this is perhaps somewhat analogous to the demand of complete evidence in inductive reasoning). But in this case, it is plausible that if the conclusion entails the conjunction of the premisses then any action in compliance with the conclusion should be taken to be in compliance with the premisses. For consider a circumstance, like the button example above, in which this might appear not to be so. Pressing buttons A and B is in compliance with the conclusion but not in compliance with the premiss 'Press button A or press buttons A, B and C'. However, this can only be thought to be objectionable because we have not fully stated which imperatives are in force. The imperative 'Press button A without button B or press buttons A, B and C' is presumably also in force and, in this case, the pressing of button A without B will be in compliance with the premiss but no part of an action in compliance with the conclusion.

All the same, the notion of strict logical equivalence is of some independent interest and, given that it and some other logical notions are not definable in terms of entailment, this then provides additional impetus to study them on their own account.

Let us note, finally, that we might introduce modal counterparts of the previous mereological notions of validity. Thus we might say that the content X *modally subsumes* the content Y if each action a in X necessitates an action b in Y, i.e. if is impossible that a is performed and b not performed (or alternatively, to adopt a somewhat weaker condition, if X necessitates Y in the sense that it is impossible that an action in X is performed while none of the actions in Y are performed). We might then say that the imperative inference X/Y is (*modally*) *valid* if X subsumes Y and Y subserves X.

The weak version of the modal criterion will lead to different results from the previous mereological criterion. Thus an inference of the form $\alpha/\alpha \vee (\alpha \wedge \beta)$ (as in 'post the letter, so post the letter or post the letter and burn down the building') will be valid under the weak modal criterion but not under the mereological criterion. The strong version of the modal criterion will, of course, lead to the same results as the mereological criterion as long as the relation of whole to part coincides with the relation of necessitation but otherwise they may differ. Thus the imperative 'Square the circle' will entail 'Find a counter-example to Fermat's Last Theorem' under the modal criterion but not under the mereological criterion.

§6 Some Systems of Imperative Logic (Incomplete)

We shall present some sound and complete axiom systems for equivalential and implication versions of imperative logic, with or without the constant $\top$. Proofs of soundness and completeness are omitted, though they can be established through a variant of the method of disjunctive normal forms developed in Fine [2015].

Take the formula $X \leftrightarrow Y$ to be valid if $X = Y$ in any model. The following axioms and rules then determine the valid $\top$-free formulas of the form $X \leftrightarrow Y$:

E1 $X \leftrightarrow \neg\neg X$
E2 $X \wedge Y \leftrightarrow Y \wedge X$
E3 $(X \wedge Y) \wedge Z \leftrightarrow X \wedge (Y \wedge Z)$
E4 $X \leftrightarrow X \vee X$
E5 $X \vee Y \leftrightarrow Y \vee X$
E6 $(X \vee Y) \vee Z \leftrightarrow X \vee (Y \vee Z)$
E7 $\neg(X \wedge Y) \leftrightarrow (\neg X \vee \neg Y)$
E8 $\neg(X \vee Y) \leftrightarrow (\neg X \wedge \neg Y)$
E9 $X \wedge (Y \vee Z) \leftrightarrow (X \wedge Y) \vee (X \wedge Z)$
E10 $X \leftrightarrow Y \,/\, Y \leftrightarrow X$
E11 $X \leftrightarrow Y, Y \leftrightarrow Z \,/\, X \leftrightarrow Z$
E12 $X \leftrightarrow Y \,/\, X \wedge Z \leftrightarrow Y \wedge Z$
E13 $X \leftrightarrow Y \,/\, X \vee Z \leftrightarrow Y \vee Z$

If the verum constant $\top$ is added to the language, we then require the addition of the following two axioms:

E14 $\top \wedge X \leftrightarrow X$
E15 $\bot \wedge X \leftrightarrow \bot$.

Let us now take the formula $X \rightarrow Y$ to be valid if X analytically entails Y in any model. The following axioms and rules then determine the valid $\top$-free formulas of the form $X \rightarrow Y$ (where $X \leftrightarrow Y$ now indicates the pair of formulas $X \rightarrow Y$ and $Y \rightarrow X$):

A1 $X \leftrightarrow \neg\neg X$
A2 $X \wedge Y \rightarrow Y \wedge X$
A3 $(X \wedge Y) \wedge Z \rightarrow X \wedge (Y \wedge Z)$
A4 $X \leftrightarrow X \vee X$
A5 $X \vee Y \rightarrow Y \vee X$
A6 $(X \vee Y) \vee Z \leftrightarrow X \vee (Y \vee Z)$
A7 $\neg(X \wedge Y) \rightarrow (\neg X \vee \neg Y)$
A8 $\neg(X \vee Y) \leftrightarrow (\neg X \wedge \neg Y)$
A9 $X \wedge (Y \vee Z) \leftrightarrow (X \wedge Y) \vee (X \wedge Z)$
A10 $X \wedge Y \rightarrow X$
A11 $X \rightarrow Y, Y \rightarrow Z \,/\, X \rightarrow Z$
A12 $X \rightarrow Y \,/\, X \wedge Z \rightarrow Y \wedge Z$
A13 $X \rightarrow Y \,/\, X \vee Z \rightarrow Y \vee Z$

If the verum constant $\top$ is added to the language, we then require the addition of the following two axioms:

A14 $X \rightarrow \top$

E15 $\bot \rightarrow \rightarrow$.

According to the first, every imperative entails the null imperative and, according to the second, the full imperative entails every imperative.

There would be some interest in developing systems for some of the other logical notions that have been considered.

§7 Some Special Inferences

I should like to discuss some cases of valid and invalid imperative inference of special interest.

Ross

The inference from α to $\alpha \vee \beta$ (as with'post the letter, so post the letter or burn the house down') is not valid under the present semantics, since an action in compliance with β may not be contained in an action in compliance with α. Compliance with the conclusion is *necessary* for complying with the premiss but not a necessary *means* for complying with the premiss.

In this regard, there is a striking connection with the notion of analytic entailment. Angell [1977] and Parry [1933]were concerned to block the argument from $p \wedge \neg p$ to q by blocking the inference from p to $p \vee q$. The definition of analytic entailment appropriate for Angell's system (Fine [2015]) turns out to be a slight variant of the current notion of entailment for imperatives; and we thereby effect a remarkable synthesis of two distinct traditions within which the principle of disjunctive weakening has been challenged.

We should note that just as the inference from α to $\alpha \vee \beta$ is invalid according to the semantics, so is the inference from $\neg\alpha$ to $\neg(\alpha \wedge \beta)$, given the equivalence between $\neg(\alpha \wedge \beta)$ and $(\neg\alpha \vee \neg\beta)$. Perhaps, from an intuitive standpoint, this is not so clear. Thus we have some inclination to accept the inference from 'Do not press button A' to 'Do not press button A and button B' even though we have no inclination to accept the inference from 'Do not press button A' to 'Do not press button A or do not press button B'. However, we also have a strong inclination to accept the inference from 'Do not press button A and button B' to 'Do not press button A or do not press button B'. And so our various inclinations seem to be in conflict.

A related issue arises with the Hare's puzzle above. As we have mentioned, imperative premisses of the form $\neg\alpha$ and $\alpha \vee \beta$ (as with 'Don't help your father', 'Help your father or mother') somehow seem contradictory. But premisses of the form α and $\neg(\alpha \wedge \beta)$ (as with 'Press button A', 'Don't press button A and B') do not. But $\neg(\alpha \wedge \beta)$ appears to entail $\neg\alpha \vee \neg\beta$ ('Don't press button A and B' entails 'Don't press button A or don't press button B').

It is not clear to me what to make of the intuitive data in these cases and of whether, in particular, there is a sensible view which sanctions the inference from $\neg\alpha$ to $\neg(\alpha \wedge \beta)$ but does not sanction the inference from $\neg\alpha$ to $\neg\alpha \vee \neg\beta$ and hence does not sanction the inference from $\neg(\alpha \wedge \beta)$ to $\neg\alpha \vee \neg\beta$. If such a view is viable, then it will be governed by a very different semantics from the one presented here.

Disjunctive Syllogism

The inference from $\neg\alpha$ and $\alpha \vee \beta$ to β (as with 'do not shut the door, shut the door or open the window, so open the window') is not valid under the present semantics, since the fusion of an action a' in compliance with $\neg\alpha$ and of an action a in compliance with α, and hence in compliance with the premisses, may not contain an action in compliance with β.

It is rather hard to say whether the inference is intuitively valid. The premisses are inconsistent in a certain way and our intuitive judgements as to what follows in the face of such inconsistency are somewhat infirm. The inference may be rendered valid if we restrict our attention to topsy models, since then the impossible action which results from fusing a' and a will contain any action in compliance with β. However, under this restriction, the inference from $\neg\alpha$ and $\alpha \vee \beta$ to $\beta \vee \gamma$ (as with 'do not shut the door, shut the door or open the window, so open the window or burn the house down') will be valid. This is not really an instance of the Ross paradox, since it is an inference in which the premisses are already unacceptable. It is perhaps problematic in the same way as the inference from $\neg\alpha$ and α to β and can, with as much justice, be either rejected or retained.

An alternative proposal is to apply the 'consistency filter'. The *filtered content of* an imperative X will be the subset of the possible actions that it allows (and, similarly, for disallowed actions). The notion of entailment might then be restricted to the filtered content. Thus, with filtering in place, X will *entail* Y if either (a) no possible action is in compliance with X or (b)(i) every possible action in compliance with X contains an action (and hence a possible action) in compliance with Y and (ii) every possible action in compliance with Y is contained in a possible action in compliance with X.

The inference from $\neg\alpha$ and $\alpha \vee \beta$ to β will then be valid while the inference from $\neg\alpha$ and $\alpha \vee \beta$ to $\beta \vee \gamma$ will not be valid. However, this solution suffers from problems of its own. The inference from $(\alpha \wedge \neg\alpha) \vee \gamma$ to $\alpha \vee \gamma$, for example, will no longer be valid (since a possible action in compliance with α will not be part of a possible action in compliance with $\alpha \wedge \neg\alpha$, given that no possible action is in compliance with $\alpha \wedge \neg\alpha$).

The inference from α to $\alpha \vee \perp$ will be valid under the operation of the consistency filter though not valid under the restriction to topsy models, since the action $\blacksquare$ in compliance $\perp$ may not be contained in any action in compliance with α. Thus the two solutions are incomparable in their results. A third solution, more comprehensive than the two others, replaces condition (a) in definition of entailment above with the weaker condition that the premiss X be weakly unsatisfiable, i.e. that some impossible action is in compliance with X. Thus any kind of 'contamination' in the premisses will lead to an inconsistency of the worst kind, one in which everything then follows. This weaker condition might be justified on the grounds that the actions in compliance with an imperative should be permitted and no impossible action can be permitted.

Under this proposal, the inference from $\neg\alpha$ and $\alpha \vee \beta$ to *any* conclusion γ will be valid. Thus on this proposal, in contrast to all the other definitions of validity we have so far considered, an inference may be valid without being classically valid. Despite this oddity, there is, I believe, a great deal to be said in its favor, especially if one wishes to adopt a liberal conception of inference under which any conclusion whatever will follow from inconsistent premisses.

Idempotence

As we have noted, the inference from α to $\alpha \wedge \alpha$ (as with 'Take a prize. So take a prize and take a prize') is not valid under our semantics. It is rather hard to evaluate the intuitive validity of the inference since the repetition of what one has already said, unless by way of emphasis or the like, is usually regarded as anomalous.

However, there is some evidence that the inference should not be regarded as valid in ordinary language (and that α should not be regarded as equivalent to $\alpha \wedge \alpha$). For consider the universal imperative 'For each win, take a prize'. This might be symbolized in the form '$\forall$w(take a prize)', where the quantifier '$\forall$w' ranges over wins; and it is naturally taken to be tantamount to the conjunction of its instances. Suppose then that there are two wins, w_1 and w_2. It will then be tantamount to 'Take a prize and take a prize' (since the quantification is vacuous). But the original universal imperative allows one to take two prizes and so the self-conjunction 'Take a prize and take a prize' will also allow one to take two prizes and will not be entailed by the conjunct 'Take a prize'.

§8 Antecedents and Comparisons

I wish briefly to discuss the related work of some other authors. The work may relate either to the underlying semantics or to the subsequent definition of validity and, in each case, may relate principally to indicatives or specifically to imperatives. Unfortunately, a comprehensive and detailed comparison is beyond the scope of the paper.

We begin with the semantics. Van Fraassen [1969], as far as I am aware, was the first published statement of a truthmaker semantics for propositional logic (references to the subsequent history can be found in Fine [2016a]). The truthmaker approach bears some resemblance to inquisitive semantics (as in Ciardelli et al [2013]) and 'alternative'-based semantics (as in Alonso-Ovalle [2006] and Aloni [2007]), for all three approaches take seriously the idea that there may be relevantly different ways, within a given possible world, in which a sentence may be true.

We find something like the application of truthmaker semantics to imperative or deontic sentences in Stelzner [1992], van Rooij [2000], Mastop [2005] and Alonso & Ciardelli [2013]. But in all of these cases, going all the way back to van Fraassen [1969], one finds that the authors adopt what is, at best, a very particular realization of the truthmaker approach. For the 'states', or candidate verifiers, are either identified with sets of possible worlds or the like (resulting in an 'intensional' state space) or with sets of literals or the like (resulting in a 'canonical' state space). One also finds that the definition of 'model' is not sufficiently general, in that the atomic sentences of the language are provided with a special kind of semantic value with the consequence that the resulting set of valid formulas is not closed under substitution.

In some of this other work, there are also differences in the form of clause adopted for some of the connectives. I should mention, in particular, the treatment of negation in Alonso & Ciardelli [2013]. Suppose that a_1, a_2, ... are the actions in compliance with the imperative X, which we may identify with subsets of the set W of possible worlds. Alonso and Ciardelli then take there to be a single action, W - $(a_1 \cup a_2 \cup ...)$, in compliance with X. So the single action in compliance $\neg\neg$X will be W - (W - $(a_1 \cup a_2 \cup ...)$) = $(a_1 \cup a_2 \cup ...)$. The double negation $\neg\neg$X, in effect, flattens the meaning of X, substituting a single disjunctive action $(a_1 \cup a_2 \cup ...)$ (the performance of one of a_1, a_2, ...) in place of the individual actions a_1, a_2, ...

Quite apart from questions of empirical adequacy, this move, to my mind, destroys one of the principal rationales for adopting the truthmaker approach. For if we think of an action as an arbitrary set of possible worlds, then imperative or deontic sentences can longer serve as a guide to action. To use an example discussed at length in Fine [2014], suppose God tells Eve: eat infinitely many of apples from Alternative Eden (E). Then E is logically equivalent to E′: eat infinitely many apples from Eden or Alternative Eden. So is she allowed to eat the Forbidden Fruit? There is no way of telling. This problem disappears if the actions taken to be in compliance with E are taken to be different from the actions in compliance with E′, as they are under the truthmaker approach, where the single logical equivalent is allowed to resolve, so to speak, into different ranges of choice. But the problem reappears once we are allowed to flatten the meaning of the embedded clause.

We turn to the concept of validity. As I have mentioned, the present notion of validity has application to Angell's system of analytic implication and some relevant citations are given in Fine [2015]. Related notions of validity, in application to the logic of imperatives, can be found in in Stelzner [1992], van Rooij [2000], Aloni [2007], and Aloni & Ciaredelli [2014]. However, in all of these cases there are a number of discrepancies from the definition given here, quite apart from general issues of presentation. So, for example, Stelzner's analogues of the subsumption and subservience conditions are somewhat weaker than our own; van Rooij's conditions are relativized to a world; Aloni presents a modal form of the definition; and Aloni and Ciardelli convert the alternative actions presented by an imperative into exclusive options.

It should also be mentioned that something like the second component condition of subservience is sometimes to be found on its own as a criterion of validity. Thus the notion of p-entailment from Kamp [73] loosely corresponds to subservience (though Kamp only has statements of permission in mind, not imperatives in general). Or again, Kenny [1966] and Geach [1966], following him, takes imperative to involve reasoning from means (as stated in the conclusion) to ends as (stated in the premisses). However, the means for Kenny are logically sufficient conditions whereas, for us, they are partially constitutive conditions.

Appendix

Let me present a brief formal exposition of the proposed semantics. I give definitions only and do not state any results.

An *imperative formula* is one constructed in the usual way from the imperative atoms α_1, α_2, ... and the constant $\top$ by means of the connectives, $\neg$, $\wedge$ and $\vee$. We use α, β, γ and the like for arbitrary imperative atoms and X, Y, Z and the like for arbitrary imperative formulas.

An *action space* $\boldsymbol{A}$ is a structure of the form $(A, \sqsubseteq)$, where A (actions) is a set and $\sqsubseteq$ (part-whole) is a partial order on A for which each subset B of A has a least upper bound $\bigsqcup B$. We use obvious notation in connection with an action space and, in particular, use $\square = \bigsqcup \varnothing$ for the *null* action and $\blacksquare = \bigsqcup A$ for the *full* action.

A *modalized action space* $\boldsymbol{A}$ - or M-*space*, for short - is a structure of the form $(A, A^\Diamond, \sqsubseteq)$, where $(A, \sqsubseteq)$ is an action space and $A^\Diamond$ (possible actions) is a non-empty subset of A subject to the conditions that $b \in A^\Diamond$ whenever $a \in A^\Diamond$ and $b \sqsubseteq a$ and that $\blacksquare \notin A^\Diamond$. We say that an action a in an M-space is *consistent* or *possible* if $a \in A^\Diamond$ and *inconsistent* or *impossible* otherwise. A subset B of actions is said to be *compatible* if their fusion belongs to $A^\Diamond$ and to be *incompatible* otherwise.

An (*action space*) *model* $\boldsymbol{M}$ is an ordered triple $(A, \sqsubseteq, |\cdot|)$, where $(A, \sqsubseteq)$ is an action space and $|\cdot|$ (valuation) is a function taking each atom α into a pair (C, C') of non-empty subsets of A. When $|\alpha| = (C, C')$, we let $|\alpha|^+$ or $|\alpha|$ be C (the set of actions in compliance with α) and let $|\alpha|^-$ be C' (the set of actions in contravention of α).

A *classical model* $\boldsymbol{M}$ is an ordered quadruple $(A, A^\Diamond, \sqsubseteq, |\cdot|)$, where $(A, A^\Diamond, \sqsubseteq)$ is an M-model and, for each atom α, (i) no member of $|\alpha|^+$ is compatible with a member of $|\alpha|^-$ and (ii) each possible action is compatible with a member of $|\alpha|^+$ or of $|\alpha|^-$.

Given a model $\boldsymbol{M} = (A, \sqsubseteq, |\cdot|)$, an action $a \in A$ and an imperative X, we may give an inductive definition of what it is for a to *comply with* X (a ||- X) or to *contravene* X (a -|| X):

(i)$^+$ a ||- α if $a \in |\alpha|^+$;
(i)$^-$ a -|| α if $a \in |\alpha|^-$;
(ii)$^+$ a ||- $\top$ if $a = \square$
(ii)$^-$ a -|| $\top$ if $a = \blacksquare$
(ii)$^+$ a ||- $\neg$X if a -|| X;
(ii)$^-$ a -|| $\neg$X if a ||- X;
(iii)$^+$ a ||- X $\wedge$ Y if for some b and c, b ||- X, c ||- Y and $a = b \sqcup c$;
(iii)$^-$ a -|| X $\wedge$ Y if a -|| X or a -|| Y;
(iv)$^+$ a ||- X $\vee$ Y if a ||- X or a ||- Y;
(iv)$^-$ a -|| X $\vee$ Y if for some b and c, b -|| X, c -|| Y and $a = b \sqcup c$.

Relative to a model $\boldsymbol{M} = (A, \sqsubseteq, |\cdot|)$,, we let $|X|^+ = |X| = \{a \in A$: a ||- X$\}$ and $|X|^- = \{a \in A$: s -|| X$\}$; and we may write $|X|^+$ or $|X|$ as X^+ or X and $|X|^-$ as X^-. For an M-model $\boldsymbol{M} = (A, A^\Diamond, \sqsubseteq)$, we let the *filtered content* $|X|^\Diamond$ (or $X^\Diamond$) *of* X be $\{a \in A^\Diamond$: a ||- X$\}$.

Given an action space $\boldsymbol{A} = (A, \sqsubseteq)$ and subsets X and Y of A, we say:

(i) *X subsumes Y* if for each $a \in X$ there is a $b \in Y$ for which $a \sqsupseteq b$;
(ii) *Y subserves X* if for each $b \in Y$ there is an $a \in X$ for which $a \sqsupseteq b$;
(iii) *X entails Y* if X subsumes Y and Y subserves X.

More generally, given a well-ordered sequence $X_1, X_2, \ldots$ of subsets of A, set $X_1 \sqcup X_2 \sqcup \ldots = \{a_1 \sqcup a_2 \sqcup \ldots: a_1 \in X_1, a_2 \in X_2 \ldots\}$. We then say that the imperative formulas $X_1, X_2, \ldots$ *entail* Y if $X_1 \sqcup X_2 \sqcup \ldots$ entails Y.

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