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Philosophy, Mathematics, Linguistics: Aspects of Interactions

**Tutorial Graham Priest / Friederike Moltmann:**

**Existence, Nonexistence, and Numbers**

**Handout 4:**

# *The Number Two* as a Reifying Term

## 1. Reifying terms

(1) a. the fictional character Hamlet.

b. the color green

c. the name John

d. the direction nborth

e. the truth value true

f. the number two

g. the numeral two

obligatory definite article

reifying sortal

attributive complement: nonreferential (occurrence ) of an expression

** the number that number, * the numeral the word I wrote down*

## 2. Differences between simple numerals and reifying terms for numbers

Numerals as noun modifiers:

(2) a. Eight women were invited.

b. The eight women were invited.

Interchangeability of numerals with reifying term:

(3) a. Eight is divisible by two.

b. The number eight is divisible by two.

(4) a. John added two to four.

b. John added the number two to the number four.

three classes of predicates that the wto kinds of terms do not generally share:

1. nonmathematical predicates
2. mathematical predicates
3. predicates describing agent-related mathematical operations

Nonmathematical predicates

(5) a. ?? Twelve, which interests me a lot, is an important number in religious and cultural contexts.

b. The number twelve, which interests me a lot, is an important number in religious and cultural contexts.

(6) a. ?? Twelve, which I like to write my dissertation about, is an important number in religious and cultural contexts.

b. The number twelve, which I like to write my dissertation about, is an important number in religious and cultural contexts.

(7) a. ?? twelve, which I thought about a lot, ...

b. the number twelve, which I thought about a lot, ....

(8) a. ?? twelve, which God created on the third day / whose significance in geometry is well-known, ...

b. ?? the number twelve, which God created on the third day / whose significance in geometry is well-known, ...

(9) a. Twelve, which is divisible by two, is not a prime number.

b. Twelve, which is smaller than fifteen, is greater than ten.

(10) a. twelve, which is a number that interests me a lot, ...

b. twelve, which is a number I like to write my dissertation about, ....

c. twelve, which is a number I thought about a lot, ....

(11) a. ?? Twelve is what I like to write my dissertation about.

b. Twelve is a number I like to write my dissertation about.

c. ?? I like to write my dissertation about twelve.

d. I like to write my dissertation about the number twelve.

predicates like *interest me*, *write about*, or *think about*:
intentional object-oriented predicates or more generally predicates that express relations that go beyond the mathematical context.

Mathematical properties

Interchangeability:

(12) a. Ten is divisible by five / is uneven / is smaller than twelve.
b. The number ten is divisible by five / is uneven / is smaller than twelve.

(13) a. the root / successor / predecessor of four
b. the root / successor / predecessor of the number four

not interchangeable:

(14) a. Two and two is four.
b. ?? The number two and the number two is the number four.

(15) a. Two plus two is four.
b. ? The number two plus the number two is the number four.

(16) a. Two times two is four.
b. ? The number two times the number two is the number four.

(17) a. Four minus two is two.
b. ? The number four minus the number two is the number two.

The Adjectival Strategy

(18) a. Two and two is four.
b. Two things and two things are four things.

(19) a. If there are (were) two things and two other things, then there would be four things.

Using plural logic:

(19) b. $\blacksquare(\exists xx \exists yy(P_1(xx) \,\&\, P_1(yy) \,\&\, \neg\exists z(z \leq xx \,\&\, z \leq yy) \rightarrow \forall xx \forall yy(P_1(xx) \,\&\, P_1(yy) \,\&\, \neg\exists z(z \leq xx \,\&\, z \leq yy) \rightarrow \exists ww(P_2(ww) \,\&\, xx \leq ww \,\&\, yy \leq ww \,\&\, \neg\exists z(z \leq ww \,\&\, \neg\, z \leq xx \,\&\, \neg\, z < yy))))$

Agent-related operations:

Generally both terms possible:

(20) a. John added four and two.

b. John added the number four and the number two.

(21) a. John subtracted two from four.

b. John subtracted the number two from the number four.

**3. Sketch of the semantics of reifying terms**

Reifying terms introduce entities on the basis of a nonreferential expression.

Sortal specifies various strategies for introducing an entity on the basis of a nonreeferential expression.

Sortal ensures that the object introduced 'holds" properties that correspond to true contexts in which the non-referential expression can occur (internal predication). In addition the introduced has the externally predicated properties that entities like fictional charatcetrs generally have.

Explanation why explicit number-referring terms and simple numerals are possible with agent-related mathematical operations:

agent-related mathematical operations involve both the intentionality of actions (and thus a non-mathematical aspect) and a purely mathematical function

the former licences explicit number-referring terms, the latter makes simple numerals acceptable

Licensing of simple numerals comes from the arithmetic aspect, licensing of explicit number-referring terms from the intentional aspect.

Simple mathematical predicates (even, infinite, is divisible by)

Predicates like *even* or *infinite* are generally defined in terms of a mathematical operation, requiring simple numerals. But since their content (unlike that of *plus* or *times*) is derivative with respect to that operation, the definition can equally well be given for pure numbers: the pure number *n* has (or 'holds') a property *P* just in case the numeral corresponding to *n* plays such and such a role in a particular mathematical operation in terms of which *P* is defined.

**4. Further linguistic evidence for the non-referential status of simple numerals**

**4.1. Two kinds of relative pronouns in German**

*Was* vs *der, die das*

*Was*: generally for nonreferential expressions

*Der, die, das* for referential expressions

(22) a. Hans wurder weise, was maria bereits ist.
'John became wise, which Mary already was.'
b. Hans hat die Eigenschaft, weise zu sein, die / * was Maria auch hat.
'John has the property of being wise, which Mary has too.'

(23) a. Hans glaubt, das es regnet, was / * das Maria auch glaubt.
'John believes that it will rain, which Mary believes too.'
b. Hans glaubt die Proposition, dass es regnet, die / * was Maria auch glaubt.
'John believes the proposition that it will rain, which Mary believes too.'

(24) a. zwoelf, was / * das / *? die eine Zahl ist, die mich sehr interessiert, …
'twelve, which is a number that interests me a lot, …'
b. zwoelf, was / * das / * die durch zwei teilbar ist, …
'twelve, which is divisible by two, …
c. die Zahl zwoelf, die / * was durch zwei teilbar ist, …
'the number twelve, which divisible by two, …'

**4.2. Support of plural anaphora**

(25) a. Hans addierte zehn und zwanzig. Maria addierte ?? sie / ok diese Zahlen auch.
'John added ten and twenty. Mary added them / those numbers too.'
b. Hans addierte die Zahlen zehn und zwanzig. Maria addierte sie auch.
'John added the numbers ten and twenty. Mary added them too.'

(26) a. Hans notierte zehn und zwanzig. Maria notierte ?? sie / ok diese Zahlen auch.
'John wrote down ten and twenty. Mary wrote them / those numbers down too.'
b. Hans notierte die Zahlen zehn und zwanzig. Maria notierte sie auch.
'John wrote down the numbers ten and twenty. Mary wrote them down too.'