

Introduction

Massimiliano Carrara and Friederike Moltmann

Plurality is an important phenomenon in natural language as well as in our thought. Typical sentences with plurals are those below:

- (1) a. The students gathered.
- b. The students slept.
- c. The students lifted the box.

Definite plural noun phrases like *the students* do not stand for single objects, but for pluralities of objects. As such, they allow for various sorts of plural predicates: collective predicates as in (1a), predicates that distribute over the individuals making up the plurality as in (1b), and predicates allowing for both a collective and a distributive interpretation as in (1c). In addition to definite plurals that stand for particular pluralities, natural language displays plural quantification, as below:

- (2) a. Some students gathered.
- b. Most students gathered.
- c. Ten students lifted the box.

Plurals pose significant challenges for logic, philosophy, and linguistic semantics. The challenge for philosophical logic consists in developing a logic containing plural terms, plural quantifiers, and plural predicates that allows deriving the intuitively valid inferences with sentences containing plurals. The challenge for philosophy consists in making sense of pluralities metaphysically. Are pluralities single entities or should they be viewed as irreducible multitudes; that is, are they “collections as one” or “collections as many” (to use Russell’s terms)? Related to that is the question of what distinguishes the referents of *the students* from that of *the group of students*? Pluralities also appear to differ from quantities, the sorts of things that mass noun phrases like *the rice* stand for, and here the question is how to make sense of the difference in semantic value of the plural term *the rice grains* and the mass term *the rice*. Plurals pose a great range of challenges for linguistic semantics. One of them is specifying the meaning of plural nouns, plural quantifiers such as *some* and *most*, and number words such as *ten*. Another challenge consists in developing a compositional semantics for sentences containing plurals and capturing the similarities and differences with respect to sentences with singular count and mass noun phrases. Finally, linguistic semantics is

faced with a great range of further phenomena associated with plurals, such as restrictions and conditions associated with plural predicates, plural anaphora, reciprocals, plural-specific modifiers such as *individual*, and conjunction with plurals, to name just a few.

There are two fundamentally different approaches to the logic and semantics of plurals: *the singularist approach* and *the plural reference approach*. The singularist approach takes definite plurals like *the students* to stand for single collective entities, mereological sums, sets, or classes. The plural reference approach takes *the students* to plurally refer to each student at once. The singularist approach has been by far the dominant one in linguistic semantics, due to influential papers by Link and Sharvy in the early eighties, and it has been further pursued by Landmann, Moltmann, Schwarzschild, and others. By contrast, logicians and philosophers have generally preferred the plural reference approach, mainly due to important papers by Boolos in the eighties. Thus the plural reference approach has been pursued by Linnebo, Rayo, Oliver/Smiley, and McKay, among others.

This volume brings together new contributions from logic, philosophy, and linguistics that address the various challenges posed by plurals and in particular the tension between the singularist and the plural reference approach to plurals. A good part of the contributions to this volume investigate and extend new perspectives presented by the plural reference approach or explore its applications to phenomena in natural language. The other contributions explore the view of pluralities as mereological sums, the relation between mass quantification and plural quantification, the semantics of plural morphology, and conditions on plural predication. In order to better situate the contributions in this volume, let us go into some further detail regarding the history of work on plurals in philosophical logic and in linguistic semantics, and regarding the various issues that are at stake.

Pluralities in logic

The aim of a logic for plurals is to develop a formal language together with a semantic interpretation that permits deriving intuitively valid inferences with statements about pluralities.

An interpreted formal language of first-order logic contains first-order variables and terms whose semantic values come from the relevant domain of individuals, that is, the domain of discourse. This means that first-order logic is just about individuals. The question then is what should a formal language look like that is also about pluralities of individuals?

One answer is that such a formal language should contain second-order variables, variables ranging over properties, sets, or, in general, pluralities of objects in the domain of discourse. While first-order variables range over a given domain of individuals and first-order logic allows us to talk about one object at a time, second-order variables range—at least on the usual interpretation—over all subsets of the domain, and, in general, higher-order logics allow us to talk about many objects simultaneously.

Traditionally, in a language L of second-order logic, we introduce symbols for predicative second-order variables X (and for functional variables u). In such a language one can quantify, universally or existentially, with first-order and second-order variables. One result is that, while in a first-order language we can simply assert that two individuals possess the same property with the formula

$$Pa \ \& \ Pb,$$

in a second-order language we can assert that any two individuals share at least one property:

$$\forall x \forall y \exists X (Xx \ \& \ Xy).$$

Historically, quantification over properties, rather than objects has generally been recognized as an important tool. For instance, Leibniz’s principle of Identity of Indiscernibles can naturally be cast within a second-order language as follows:

$$\forall x \forall y (\forall X (Xx \leftrightarrow Xy) \rightarrow x = y).$$

Also, Russell’s example “Napoleon had all the properties of a great general” has a logical form in a second-order language:

$$\forall X (\forall y (Gy \rightarrow Xy) \rightarrow X(n)).$$

Unfortunately, it is now generally admitted among philosophers that, following Quine, “second-order logic is a wolf in sheep’s clothes”; that is, second-order logic is set theory in disguise. Thus, Quine argued that second-order logic is not really a logic, but set theory, and that to study second-order logic amounts to studying the standard model of set theory.

More recently, an alternative to second-order logic with its usual interpretation, namely *plural logic*, has been explored by a number of philosophical logicians. Plural logic permits formalizing basic plural sentences by taking *plural reference* and *plural predication* to be primitives (Boolos (1984), Linnebo (2003, 2012), Oliver and Smiley (2004, 2006, 2013), Yi (2005, 2006), McKay (2006)).

Plural logic originates in Boolos’s (1984, 1985) work, though the idea of plural reference may go back much further in the history of philosophy and can be found already on Plato’s distinction between “plural subjects” and “related subjects,” as Scaltsas argues in his contribution to this volume, Boolos proposed a reinterpretation of second-order monadic logic in terms of *plural quantification*. He argues that Quine’s slogan—“to be is to be the value of a variable”—does not entail that the value of a second-order variable must be a set (or a property) of individuals. Instead, Boolos claims, the value of such a variable can be considered a *manifold* of individuals, a “class as many.” For the purpose of such a *plural interpretation* of variables, Boolos restates the Tarskian truth definition for second-order logic by modifying the notion of *assignment*.

Given that interpretation, second-order monadic logic is ontologically innocent, that is, it does not entail any commitment to classes, properties, or pluralities, but only

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to individuals. According to Boolos, second-order quantification differs from first-order quantification in that the former permits plural reference to individuals—that is reference to several individuals at once—while the latter permits only singular reference.

Boolos's basic idea consists of interpreting the atomic formulas of the form Xy , as "y is one of the Xs." Given this, an existential formula of the form

$$\exists X \dots Xy \dots$$

is to be understood as:

"There are some Xs such that . . . y is one of the Xs . . ."

Boolos gives no explanation of how to refer to an arbitrary plurality of individuals. He makes use of primitive plural existential quantification not further analysing the locution:

"There are some objects such that . . ."

which, though, is ambiguous. There are contexts where its meaning is the same as that of the first-order quantifier; in others it is not reducible to first-order quantification, as for example in the Geach-Kaplan's sentence:

(3) Some critics admire only one another.

Boolos's interpretation of second-order logic, although very attractive, has faced criticism from several philosophers (Resnik 1988, Parsons 1990, Linnebo 2003). The main "Quinian" criticism consists in the suspicion that speaking of pluralities of individuals is just a rough manner of speaking of sets. This, for example, is the essence of Parsons' criticism of Boolos in (1990). Quine's slogan that second-order logic is "set theory in disguise" (or metaphorically "a wolf in sheep clothing") does not seem to have lost all its advocates.

One question to pursue therefore is, is Boolos's plural logic really a logic? A tentative answer to it is given in Bocconi, Carrara, and Martino's contribution to this volume, where an interpretation of second-order logic, as an alternative to Boolos's proposal, is provided (on this alternative proposal, see also Carrara and Martino 2010). A second question to pursue is, are there ways to overcome the criticisms of Boolos's proposal, adopting a different approach to plural quantification? Oliver and Smiley's contribution addresses this question, exploring different ways of developing and amending Boolos's original proposal.

Linnebo's contribution to this volume concerns Frege's Basic Law V and plural logics. The law says that two open formulas $\phi(u)$ and $\psi(u)$ define the same extension when they are true of the same objects. As Russell discovered, Basic Law V allows us to derive a version of Russell's paradox and is thus inconsistent. However, Linnebo observes that some useful principles for higher-order logics collapse "higher-order entities" to single objects and to accept them "amounts to endorsing something like Basic Law V, whose function is precisely to collapse many objects into a set or a

property” (Linnebo, this volume p. 21). The aim of Linnebo’s paper is to show that Basic Law V can be harnessed, by adopting a modal framework.

A second approach to pluralities, which philosophers have defended as an alternative to Boolos’s approach and plural logic, is based on mereology, the theory of the relation of parts to a whole (for an introduction, see Simons 1987). On that view, pluralities are conceived as mereological sums in the sense of extensional mereology (where “extensional” means there are no distinct objects with the same proper parts). But does the mereological view not raise similar issues as a set-theoretic view of pluralities by involving a new ontological commitment to sums apart from the individuals?

Lewis (1991) defends the ontological innocence of mereology for the conception of pluralities. Lewis argues that, like logic but unlike set theory, mereology should be recognized as “ontologically innocent.” In other words, given certain objects, no further ontological commitment is required for the existence of their sum or their plurality. Formally, Lewis does so by extending a language of first-order logic so as to allow for singular and plural reference as well as singular and plural quantification. Such a language includes both plural terms and variables (symbolically “X”), plural quantifiers (e.g. “there are some things . . . such that”) and the special two-place predicate “. . . is one of . . .” The latter admits a singular term in its first place and a plural one in its second place. Finally, by adding to this vocabulary the non-logical predicate, “. . . is a part of . . .,” Lewis obtains a language rich enough to formulate mereology, allowing to define sums—or fusions or pluralities—and the overlapping relation.

Lewis’s argument for the innocence of mereology can be summarized as follows:

- (P1) Composition—a many-one relation—is like identity.
- (P2) The commitment to sums is already made by the acceptance of the objects that are to form sums.
- (P3) Nothing could be considered more ontologically innocent than the request to accept something identical to things already accepted.
- (P4) No other entities beyond sums of individuals are introduced in mereology.
- (C) Mereology is ontologically innocent.

By contrast, within set theory, given certain objects, the existence of the set of them requires a further ontological commitment. A set—unlike a sum—is an abstract entity whose existence is not directly entailed by the existence of its members.

Lewis himself claims that the innocence of mereology is different from that of plural reference. In the case of plural quantification, “we have many things, in no way do we mention one thing that is the many taken together.” Instead, in the mereological case, “we have many things, we do mention one thing that is the many taken together, but this one thing is nothing different from the many” (1991, 87).

Because Lewis explicitly uses sums as objects in their own right, Lewis’s innocence thesis must be understood in the sense that, even if the sum of the Xs is a well-determined object, distinct from the Xs, the existence of such an object must be accepted by anyone who has already accepted the existence of the Xs. In other words, committing oneself to the existence of the Xs would be an implicit commitment to some other entities—

among them, the sum of the *Xs*. On the other hand, the existence of the set of the *Xs* would not be implicitly guaranteed by the existence of the *Xs*. Simons' contribution to this volume disagrees with Lewis and argues that mereology is ontologically committing. It develops instead a logic for "pluralities as many," that is, multitudes, as well as for multitudes of multitudes.

Pluralities in linguistic semantics

As already mentioned, the dominant approach in linguistic semantics is the singularist approach, which treats reference with plural terms as reference to single collective entities, collections "as one." These have been formally conceived as mereological sums as in the work of Sharvy (1980), Link (1983), and much subsequent work in linguistic semantics; as sets as in Gillon (1987) and Schwarzschild (1996); and as mereological sums within a non-extensional version of mereology in Moltmann (1997, 1998, 2005).

There is considerable attraction coming from linguistic generalizations for pursuing that approach, in particular the mereological version. The advantage of the mereological view of pluralities over the set-theoretic one is that it treats pluralities entirely on a par with individuals, rather than of being a higher type, which avoids type shifting of predicates when they apply to plurals. Moreover, it allows treating plural noun phrases on a par with mass noun phrases, if the latter are taken to stand for entities in the domain of discourse as well, namely for so-called quantities. Many predicates in fact can apply to both singular count noun phrases, plural noun phrases, and mass noun phrases, as indicated below:

- (3) a. The stone is grey.
 b. The students are grey.
 c. The material is grey.
- (4) a. The stone is heavy.
 b. The stones are heavy.
 c. The material is heavy.

With plurals, *grey* displays an obligatory distributive reading, whereas *heavy* displays both a distributive and a collective reading.

Note that the same predicates can also act as noun modifiers (*the successful student/students, the heavy stone/stone*), which gives another reason to avoid type-shifting.

A further parallel between singular count noun phrases and plural noun phrases consists in that there are determiners and quantifiers that may apply to both singular count nouns and plural nouns, namely in English *the, some, and no*:

- (5) a. the child/some child/no child
 b. the children/some children/no children
 c. the wood/some wood/no wood

This has motivated Sharvy's (1980) influential, unified account of definite noun phrases, according to which definite noun phrases in general refer to the sum of the extension of the noun they apply to. In (4a), this is the (trivial) sum of the singleton set that is the extension of *stone* (in the context). In (4b), this is the sum of the extension of *stones* (in the context), the set of the mereological sums of the contextually relevant students. Finally in (4c), this is the sum of the extension of *material* (in the context), the set of relevant quantities of wood.

Yet another parallel consists in that the partitive construction is applicable to definite singular count noun phrases, definite plural noun phrases, as well as definite mass noun phrases:

- (5) a. Some of the chair is red.
 b. Some of the balls are red.
 c. Some of the wood is red.

The partitive construction appears to presuppose part-whole relations applying to individuals, pluralities, as well as quantities. Thus, *some of* appears to quantify over parts of the chair in (5a), over parts of the maximal plurality of balls in (5b), and over the parts of the maximal quantity of wood in (5c).

Also, conjunction applies, it appears, with the same interpretation to definite singular count, plural and mass noun phrases, leading to a term referring to the mereological sum of the referents of the conjuncts:

- (6) a. The man and the woman met.
 b. The men and the women met.
 c. The meat and the sauce were served separately.

Thus plural noun phrases appear semantically on a par with singular count and mass noun phrases, taking as semantic values entities of the very same type as individuals and quantities, and thus contribute in the same way to the semantics of sentences containing plurals as singular count or mass noun phrases contribute to the semantics of sentences containing singular count or mass noun phrases.

Plural nouns and mass nouns share further semantic properties, not shared, though, by singular count nouns. Both plural nouns and mass nouns have a cumulative extension. That is, if entities x and y are in the extension of a plural or mass noun, then so is the sum of x and y . Moreover, both plural nouns and mass nouns allow for proportional quantifiers like *most*, *few/little* and *many/much*. The difference between mass nouns and count nouns resides in that the entities in the extension of plural nouns are all made up from entities in the extension of singular count nouns. Given extensional mereology, these are atoms with respect to the part relation that orders the elements in the extension of the plural noun. Alternatively, given a non-extensional view of part-whole structure such as that of Moltmann (1997, 1998), entities in the extension of plural nouns necessarily consist of integrated wholes, whereas entities in the extension of mass nouns do not. Thus, the entities in the extension of mass nouns need not be made up from atoms or integrated wholes.

Despite its appeal and popularity among semanticists, the singularist approach to the semantics of plurals has faced criticism, of just the sort that has motivated the plural reference approach to plurals in general, in particular the threat of logical paradoxes and the intuition that sentences with plurals such as *the students gathered* do not involve an ontological commitment beyond that to the individual students. Some semanticists therefore have pursued the path initiated by Boolos (1984, 1985), in particular Schein (1995) and McKay (2006). Moltmann's contribution to this volume is an extensive discussion of the singularist approach and the plural reference approach. It focuses on the empirical linguistic facts that bear on the choice among the two approaches and presents novel arguments in favor of the plural reference approach.

The plural reference approach raises the question of how it can be integrated within Generalized Quantifier Theory, the semantic theory of quantifiers commonly adopted by semanticists concerned with quantification in natural language. The question is particularly important in view of the fact that *most*—the quantifier that had motivated Generalized Quantifier Theory in the first place—is not in fact a singular quantifier, but a plural quantifier, allowing for collective predicates (as in *Most students gathered*). The question of how to combine plural quantification with Generalized Quantifier Theory is the subject matter of Yi's contribution to this volume.

The plural reference approach raises a range of further interesting issues, which, though, are not addressed in this volume. One of them is how it could be integrated within a compositional semantics of the sort of Montague Grammar, or a structured propositions framework. Since plural reference posits several semantic values for plural terms at once, it is unclear how this approach could be made compatible with a compositional semantics according to which each constituent contributes a single semantic value to the compositional semantics of the sentence. Similarly, on the standard view of structured propositions, expressions in a sentence contribute just a single semantic value to a structured proposition, and it is unclear how structured propositions and plural reference could then be combined. Such questions are yet to be addressed by formal semanticists.

Three further issues regarding plurals, though, are addressed in the present volume. One of them concerns the semantics of plural morphology. On the standard view, plural morphology goes semantically along with an operation of forming collections from the elements in the extension of singular count nouns. In his contribution to the volume, Aquaviva shows that this view is much too simplified. It is inapplicable, for example, to plurals of the sort *0, 5 apples* or lexical plurals such as *rains*. Aquaviva proposes an alternative account that pays much greater attention to theoretical and empirical aspects of plural morphology.

Another issue concerns the difference between plural and mass nouns. While current approaches to mass nouns often assimilate the semantics of mass nouns to that of plurals, making use of the very same formal notions, McKay, in his contribution to this volume, pursues an approach that takes the semantics of mass terms and mass quantifiers to be more basic than that of plural terms and plural quantifiers.

Finally, plurals raise the question of the understanding of plural predication. Sentences such as *the reporters asked questions* do not require each reporter to have asked a question, they only require partial involvement of the plurality. Arapinis's contribution to this volume examines more closely the forms and conditions on partial involvement that pluralities need to satisfy for a plural predicate to be applicable.

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