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The Number of Planets, a Number-Referring Term?

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The question whether numbers are objects is a central question in the philosophy of mathematics. Frege made use of a syntactic criterion for objethood: numbers are objects because there are singular terms that stand for them, and not just singular terms in some formal language, but in natural language in particular. More specifically, Frege (1884) thought that both noun phrases like the *number of planets* and simple numerals like *eight* as in (1) are singular terms referring to numbers as abstract objects:

(1) The number of planets is eight.

Frege took it to be obvious that (1) is an identity statement.

In this paper, I will argue that Frege's view about reference to numbers in natural language is fundamentally mistaken. *The number of planets*, I would like to show, while it in general is a referential term, is not a term referring to a number (and in fact in the particular context of (1) it is not a referential term at all). In general, *the number of planets* does not refer to an abstract object, but rather to what I will call a *number trope*, the concrete instantiation of a 'number property' in a plurality, namely the instantiation of the property of being eight in the plurality of the planets. Moreover, I will argue that (1) is not an identity statement.

6.1 THE NUMBER OF PLANETS AS A REFERENTIAL, BUT NOT A NUMBER-REFERRING TERM

Let me call terms like *the number of planets* '*the number of*-terms'. It was Frege's view that since *the number of*-terms are referential terms, they must have the function of standing for an object (Frege's context principle), and since Frege thought that only numbers could be the right objects of reference, numbers are objects. I will argue that in many (though not all) contexts, *the number of planets* has indeed the status of a referential term, but it refers to what I call a 'number trope', a particularized property which is the instantiation of a 'number property' in a plurality of entities. Thus, *the number of* *planets* will refer to the instantiation of the number property being eight in the plurality of the planets. I will use the term 'plurality' to mean whatever plural terms may stand for. Obviously this should be a collection-as-many, rather than a collection-as-one. The view that *the number of planets* refers to a number trope with a plurality as bearer is relatively independent of the particular view one may take about the semantics of plurals, though I myself will chose the view that plurals refer to several individuals at once (plural reference), rather than standing for a single collection, a plurality.[1]

There is a range of semantic evidence that indicates that noun phrases of the sort *the number of planets* (*the number of*-terms) do not refer to numbers as abstract objects. First of all, Frege's example (1) cannot be a statement of identity. Substituting the simple numeral *eight* in (1) by an explicit number-referring term results in a sentence that is much less acceptable, for the purpose of expressing the proposition that (1) expresses:[2]

(2) ?? The number of planets is the number eight.

Here and throughout the paper, '??' means 'is semantically unacceptable', that is, unsuited for the purpose of expressing the relevant kind of proposition. By contrast, '*' indicates, as is standard, ungrammaticality. Even if (2) itself might not convince everyone that (1) is not an identity statement, we will later see linguistic evidence that is rather conclusive to that effect. But if (1) is not an identity statement, what is its logical form? I will argue that (1) is neither an identity statement nor a subject-predicate sentence, but rather is of a third sort, namely what linguists call a pseudocleft or specificational sentence, a sentence where (at least on one view) the subject expresses a question and the postcopula NP an answer. *The number of*-terms, however, clearly occur as referential terms in a range of contexts, and I will now focus on those. For example, in contexts such as (3a), *the number of women* satisfies any tests of referentiality. In particular, in that sentence it occurs as subject of a sentence whose predicate generally acts as a predicate of individuals, just as in (3b):

(3) a. The number of women is small.
    b. The number eight is small.

Let me call terms like *the number eight* 'explicit number-referring terms'. Explicit number-referring terms and *the number of*-terms display a range of semantic differences with various classes of predicates as well as in other respects.

[1]The use of 'plurality' in the metalanguage, thus, is meant to functions like a plural term, rather than the collective singular noun phrase that it in fact is.

[2]This is despite Frege's own claim to the contrary (Frege, 1884). The example is equally unacceptable in German. In fact also Frege's other German example below, where the numeral occurs with a definite determiner, is unacceptable in my ears:

(1) ?? Die Anzahl der Planeten ist die Acht.

'The number of planets is the eight.'

These differences are evidence that the two kinds of terms refer to fundamentally different sorts of entities: *the number of*-terms refer to number tropes; by contrast, explicit number-referring terms refer to abstract objects, to what I will call 'pure numbers'.

6.2 PREDICATES

Most importantly, *the number of*-terms and explicit number-referring terms differ in the range of predicates they accept or in the readings they display with particular kinds of predicates. There are a number of predicates that are perfectly natural with *the number of*-terms, but are not acceptable or not as natural (for expressing the relevant sort of meaning) with explicit number-referring terms. Such predicates include *exceed* and *equal*. Thus, (4b), while grammatical and in fact meaningful, is not well-suited to express the proposition expressed by (4a) (but rather (4b) leaves open in what respect the one number is to exceed or equal the other):

(4) a. The number of the women exceeds the number of the men.
 b. ?? The number fifty exceeds the number forty.

It is significant that the same predicate, with the addition of the modifier *in number*, is acceptable with corresponding plural noun phrases, for the purpose of expressing the proposition expressed by (4a):

(4) c. The women exceed the men in number.

One-place predicates of comparative measurement in general display the same semantic pattern with *the number of*-terms, explicit number-referring terms, and the corresponding plurals, for example *negligible*, *significant*, *high*, and *low*:

(5) a. The number of animals is negligible / significant.
 b. The animals are negligible / significant in number.
 c. ?? The number 10 is negligible / significant. (different understanding of the predicate)

(6) a. The number of deaths is high / low.
 b. The deaths are high / low in number.
 c. ?? The number ten is high / low. (different understanding of the predicate)

The closeness of the referents of *the number of*-terms to the associated plurality is also revealed in the readings such terms yield with other evaluative predicates. With both kinds of entities, evaluative predicates in general do not display the kind of reading expected when applying to abstract objects, as in (7a) and (8a), but rather readings that yield an evaluation of the plurality in just one particular respect, namely with respect to how many they are, as in (7b) and (8b):[3]

[3] One might take *the number of women* in (7a) to be a 'concealed fact' (Grimshaw, 1997) rather than a term referring to an object. While this might provide an alternative explanation between

(7) a. The number of women is unusual.
 b. The number fifty is unusual.

(8) a. John compared the number fifty to the number forty.
 b. John compared the number of women to the number of men.

(7a) has quite a different reading from (7b), and (8a) from (8b). The readings that (7a) and (8a) display can be made transparent by the near-equivalence with a sentence just about the plurality such as (9a) and (9b), with a modifier 'in number':

(9) a. The women are unusual in number.
 b. John compared the women to the men in number.

Thus, unlike pure numbers, the entities that *the number of*-terms refer to share certain kinds of properties with the corresponding pluralities. These are precisely the properties that can be attributed to the pluralities when adding the modifier 'in number'. They are the properties the plurality has when viewed only as 'how many it consists in', that is, when focusing just on how many entities make the plurality up. This gives a first indication of what kinds of entities *the number of*-terms refer to: they are aspects of a plurality, concerning just how many things make up the plurality.

There are further properties that show that referents of *the number of*-terms, unlike pure numbers, are entities that are close to the associated plurality. These properties indicate that as long as the plurality consists of concrete entities, the referents of *the number of*-terms also qualify as concrete. Common criteria of an entity being concrete rather than abstract is its ability to act as an object of perception and to enter causal relations. We can then observe that as long as the plurality in question consists of concrete entities, perceptual and causal predicates make sense with *the number of*-terms, though not with explicit number-referring terms:

(10) a. John noticed the number of the women / ?? the number fifty.
 b. The number of the women / ?? The number fifty caused Mary consternation.

Of course, if the plurality is itself abstract, predicates of perception and causation are inapplicable (as with *the number of natural numbers below ten*).

6.3 NUMBER TROPES

The number of-terms thus refer to entities that have two characteristics:

[1] They share those properties with the corresponding plurality that can be attributed to the plurality with the addition of the modifier 'in number'.

(8a) and (8b), it is not applicable in general, for example not to the examples in (8). Moreover, it would not account for the properties of concrete objects that the referents of *the number of*-terms display, as discussed below.

[2] They have causal properties qualifying them as concrete, as long as the corresponding plurality consists of concrete entities.

There is one kind of entity that fits just these two roles, and this is a certain kind of trope, that is, particularized property, namely what I will call a *number trope*.

Standard examples of tropes discussed in the philosophical literature are 'Socrates's wisdom', 'the beauty of the landscape', or 'the heaviness of the vase'. Tropes of this sort behave just the same way with respect to [1] and [2]. They accept causal and perceptual predicates as below:[4]

(10) a. John noticed the beauty of the landscape.
b. The heaviness of the vase caused the table to break.

Moreover they exhibit the same pattern regarding predicates of measurement and evaluation:

(11) a. Socrates's wisdom exceeds Xanthippe's.
b. Socrates exceeds Xanthippe in wisdom.
c. The property of being very wise exceeds the property of being not very wise.

(12) a. Plato compared Socrates's wisdom to Xanthippe's.
b. Plato compared Socrates to Xanthippe in wisdom.
c. Plato compared the property of being very wise to the property of being not very wise.

Using tropes for the semantics of number terms requires a few words about the notion of a trope in general. A trope is a particularized property, a concrete manifestation of a property in an individual (the bearer of the trope). While tropes have received a particular interest in more recent metaphysics, they form an ontological category interest in which goes back as far as Aristotle. For Aristotle, tropes (or 'accidents' as they are called in Aristotelian metaphysics) were an ontological category besides individuals substances) and universals (secondary substances and qualities). In Aristotelian metaphysics, tropes are entities that are ontologically dependent on a bearer. For example, Socrates's wisdom is ontologically dependent on Socrates. Moreover tropes generally are considered the instances of universals, more precisely the instances of qualitative universals. Thus Socrates's wisdom is an instance of wisdom. While

[4] Tropes are generally taken to come with a spatio-temporal location and thus would qualify as concrete in yet another respect (at least if their bearer is concrete). However, it appears that tropes in fact resist the attribution of a location in space. This holds both for number tropes and for other tropes, as is reflected in the unacceptability of the examples below:

(1) a. ?? Mary's beauty in the room.
b. ?? Mary's weight on the bed
c. ?? the number of cards which was on the table.

Tropes should better be viewed as particulars that are ontologically dependent on an object that may have a spatial location, but that themselves are not spatially located.

qualitative universals have tropes as instances, such universals can be predicated only of the bearers of the tropes instantiating them. Thus wisdom is true of Socrates, not of Socrates's wisdom.

While tropes in this sense have played a role not just in ancient metaphysics, but throughout the middle ages, early modern philosophy (Hume, Locke, Husserl), as well as in contemporary metaphysics (Wolterstorff, 1970; Lowe, 1989, 1998), they also play a somewhat special role in a recent tradition in metaphysics initiated by the seminal paper of Williams (1953). The interest there is in a one-category ontology, with tropes being the only fundamental ontological category. According to that view, individuals are bundles of co-located tropes and universals are classes of resembling tropes.[5] The present interest in using tropes for the semantics of number-terms is entirely independent of the ambitions of such a one-category ontology; no commitment is made that universals or individuals reduce to tropes. The only claim that is made is that natural language makes reference to tropes rather than pure numbers with what since Frege was thought were number-referring terms. This paper will make use of properties without taking any stance whether or not they may be reduced to tropes.

Let us then turn to number tropes, the tropes that I argue *the number of planets* refer to. The bearer of such a trope is a plurality, the plurality of the planets (and I mean this to be a collection-as-many, rather than a collection-as-one). A number trope is a trope that consists in just one aspect of the plurality, namely its numerical aspect, which concerns just how many entities the plurality consists in. It disregards all qualitative aspects of those entities. A number trope, in other words, is the instantiation of a property of being so-and-so-many in a plurality. For example, the trope that *the number of planets* refers to will be the concrete manifestation of the property of being eight in the plurality of the planets.

A number trope differs from standard examples of tropes (such as Socrates's wisdom or the redness of the apple) in that it is purely quantitative.[6] Psychologically speaking, it involves 'abstracting' from all the qualitative respects of a plurality and focusing just on how many it consists in. Ontologically speaking, a number trope is an entity that shares only those properties of the underlying plurality that pertain to how many entities the plurality consists in. Other quantitative tropes are John's height, Mary's age, and Bill's weight.

Number tropes have still other kinds of properties than those discussed so far. In particular, number tropes display a range of mathematical properties. But first let us focus on the conception of number tropes itself and the semantics of number trope terms.

The semantics of number trope terms requires an account of plural terms such as *planets*. The main point of this paper does not hinge on the partic-

[5] For that view, see also Campbell (1990) and Bacon (1995).

[6] See also Campbell (1990) and Moltmann (2009) for the notion of a quantitative trope.

ular way of treating plurals, whether plurals stand for a single entity that is a plurality or whether they involve plural reference, referring to several individuals at once, as in plural logic (Boolos, 1984; Yi, 1999, 2005, 2006). I myself will adopt the view of plural reference. Given that view, *two* would not be a predicate holding of single objects, plural entities of some sort, but rather a predicate applicable to several individuals at once, and it would be true of several individuals just in case among them are two distinct individuals with which all the others are identical, as in (13):

(13) For entities dd, $two(dd) = 1$ iff $\exists x \exists y(x \leq dd \;\&\; y \leq dd \;\&\; x \neq y \rightarrow \forall z(z \leq dd \rightarrow z = x \vee z = y))$

In (13), 'dd' is a plural variable, that is, a variable that can stand for more than one individual at once, $\leq$ is the relation 'is one of', and 'x' and 'y' are singular variables, variables that can stand for only single individuals.

Number trope terms are formed with the unspecific functional relational noun *number*. *Number* in *the-number-of* terms expresses a plural function, a function which maps any n individuals simultaneously to the trope that is the instantiation of the property of being n in those individuals:

(14) For entities dd, $number(dd) = f(P, dd)$ for some number property P such that $P(dd)$.

Here 'f' stands for the function mapping a property or a relation and an individual or several individuals to the instantiation of the property or relation in the individual or the individuals (in case the individual(s) instantiate(s) the property or stand in the relation; it will be undefined otherwise).

(14) raises a potential problem. When generalized to arbitrary pluralities, in particular infinite pluralities, there will not be a unique number property, but rather there will be many number properties that would be true of the plurality (though not in the case of finite pluralities). Thus, considering *number* to express a function may seem problematic, unless it is a choice function, choosing one among the set of applicable number properties. However, an infinite plurality should in fact be the bearer of only a single number trope for the various number properties true of the plurality, rather than being the bearer of different number tropes for different number properties. The reason is that tropes, as has been argued, also play the role of truth makers: they ground the application of predicates to individuals (Mulligan et al., 1984; Moltmann, 2007). That is, what makes a sentence such as *John is happy* true is the particular entity in the world that is the trope of John's happiness. Given a particular infinite plurality, clearly it is one and the same actual feature of that plurality, its numerical aspect, in virtue of which it is the case that the plurality has ω-many members as well as $(\omega + 1)$-many members etc.

The semantics of *the number of planets* is then as follows, where $[planets]^{w,i}$ is the restriction of the plurally referring term *planets* to the actual circumstances, the actual world w and the present time i:[7]

(15) $[\textit{the number of planets}]^{w,i} = f(P, [\textit{planets}]^{w,i})$, for some number property P that holds of $[\textit{planets}]^{w,i}$

Note that on this view *the number of* is not a functor applying to a concept-denoting expression, as Frege assumed. In fact, a concept-denoting expression (a predicate) is impossible in that context (**the number of is a planet*, **the number of a planet*).[8]

There is one potential problem for the number trope analysis of *the number of*-terms and that is cases in which the relevant plurality is empty, as in (16):

(16) The number of students this year is zero.

But, as will be discussed in Section 6.5, (16) is in fact a specificational sentence. That is, the subject here has the function of specifying the question 'How many students are there?' and the numeral in postcopula position that of specifying an elliptical answer

Another apparent problem is identity statements as in (17):

(17) The number of women is the same as the number of men.

There is good evidence, however, that the expression *the same as* in (17) expresses not numerical identity, but rather qualitative identity or close similarity among tropes. This is also the case with other trope-referring terms:

(18) a. John's excitement today is the same as John's excitement yesterday.
b. John's irritation is the same as Mary's.
c. John's weight is the same as Bill's.

[7] One potential semantic problem with number trope terms in English is that *the number of* is actually not followed by a standard plural term, that is, a definite plural NP, but rather by a bare (that is, determinerless) plural. While there are different views about the semantic function of bare plurals, it is generally agreed that bare plurals can act as kind-referring terms (Carlson, 1977). In Moltmann (2013), I argue that bare plurals and mass nouns should themselves be considered plurally referring terms, referring plurally to the various instances in the various possible circumstances, so that *rare* would be a plural predicate. In certain contexts, such as that of the functor *the number of*, only the instances of an actualized kind are taken into account, that is, the instances of the kind when restricted to the actual circumstances, which means, the same entities that a definite plural term refers to. Note that in some languages 'the number of' can be followed by a definite plural only (or a specific indefinite), for example in German (*die Anzahl der Planeten* / **von Planeten* 'the number of the planets / of planets').

[8] *The number of*-terms may also refer to what appears to be an entity that has variable manifestations as number tropes – namely at different times or in different possible circumstances:

(1) a. The number of students has increased.
b. The number of students might have been higher than it is.

The number of students in (1a) and (1b) does not refer to a single number trope, but rather to a function-like entity, characterized by a function f mapping a world w and a time i to a manifestations that is a number trope in w at i. See Moltmann (2013, forthcoming) for discussion.

The same as in fact can express qualitative identity or similarity also with individuals:

(19) John owns the same car as Mary.

By contrast the *is* of identity can express only numerical identity and thus it does not seem quite right in the examples below:

(20) a. The number of women is the number of men.
b. John's excitement is Mary's excitement.
c. John's weight is Bill's weight.
d. John's car is Mary's car.

(20a–c) sound false, as does (20d) if John and Mary own distinct cars.

Also identity statements such as (21a), which would have to express numerical identity (or perhaps 'relative identity'), are bad. Such statements are of course fine with distinct descriptions referring to the same individual as in (21b):

(21) a. ?? The number of women and the number of men are the same number.
b. The carpenter and the professor are the same person.

6.4 MATHEMATICAL PROPERTIES OF NUMBER TROPES

Number tropes have not only the kinds of properties that are characteristic of tropes in general. They also have certain kinds of mathematical properties, though they do not share the full range of mathematical properties that pure numbers can have, that is, the referents of explicit number-referring terms. I will argue that the more limited range of properties that number tropes may have (in contrast to pure numbers) follows from the nature of number tropes itself.[9]

Let us first look at predicates that classify numbers according to their mathematical properties. Predicates such as *even*, *uneven*, *finite* and *infinite* are possible both with number tropes and with pure numbers:

(22) a. Mary was puzzled by the uneven / even number of guests.
b. Given the merely finite number of possibilities, . . .
c. John pointed out the infinite number of possibilities.

There are other predicates, however, that are semantically acceptable only with pure numbers but not number tropes. They include *natural*, *rational*, and *real*:

(23) ?? the natural / rational / real number of women

[9] Some caution is needed concerning the linguistic generalizations in this section. In a number of areas of mathematics, such as elementary combinatorics, 'the number of Xs' is explicitly defined as a pure number, the cardinality of the set of Xs. Speakers used to the mathematical literature will not share all of the data discussed in this section. But I can assure that the data have been confirmed with a large number of 'ordinary' speakers.

Furthermore, many mathematical operations are inapplicable to number tropes. These include one-place operations such as the successor function:

(24) ?? the successor of the number of planets

By contrast, the two-place functions *sum* and *plus* are applicable to number tropes:

(25) a. the sum of the number of men and the number of women
b. The number of children plus the number of adults is more than a hundred.

What distinguishes the mathematical predicates or functors that are applicable to number tropes from those that are not? The answer to this question can be obtained by reflecting on the kinds of mathematical properties concrete pluralities can have and the kinds of operations that can apply to them.

First of all, there is a sense in which pluralities can be even or uneven: to see whether a plurality is even or uneven, it just needs to be checked whether or not the plurality can be divided into two equal subpluralities. Similarly, in order to see whether a plurality is finite or infinite it simply needs to be seen whether or not a 1–1 mapping can be established from the elements of the plurality onto themselves. A number trope will then be even, uneven, finite, or infinite simply because the plurality that is its bearer is. Let us then state the following generalization: a mathematical predicate is applicable to one or more number tropes just in case its application conditions correspond to hypothetical operations on the pluralities that are the bearers of the number tropes.

Such a condition also explains the applicability of the functor sum: the sum operation is applicable to two number tropes because it can be defined in terms of an operation on the two pluralities that are the bearers of the number tropes

(26) *Addition of Number Tropes*
For two number tropes t and t', $sum(t, t') = f(P, dd)$ for some number property P and for individuals ee such that $t = f(P_1, ee)$ and individuals aa such that $t' = f(P_2, aa)$, for number properties P_1 and P_2:

$$\forall d(d \leq dd \leftrightarrow d \leq ee \vee d \leq aa), \text{ provided } \neg\exists d(d \leq ee \,\&\, d \leq aa).$$

As an operation on number tropes, the sum of tropes t and t' and the sum of tropes t'' and t''' will be distinct tropes even if t and t'' as well as t' and t''' have equinumerous bearers. But in the latter case, the sum of t' and t'' and the sum of t'' and t''' will be exactly similar and thus 'the same'.

Why isn't the successor function applicable to number tropes? The reason is simply that the successor function cannot be viewed as an operation on pluralities: the successor function as a function applying to a plurality would require adding an entity to the plurality. However, given a 'normal' universe, there is not just one single object that could be added, but rather there are many choices as to what object could be added to the plurality to yield its successor. Thus, no uniqueness is guaranteed, which means as an operation on pluralities, the successor function is just not a function (and it better not be a choice function, given that arithmetic should not as such presuppose the Axiom of Choice). Similar considerations rule out the predecessor, root, and exponent functions as operations on number tropes.

Thus we can state the condition arithmetical operations on number tropes as follows:

(27) *Condition on arithmetical properties of and functions on number tropes*

a. If P is an n-place arithmetical property of number tropes, then for some n-place property of pluralities Q, for any number tropes $t_1, \ldots, t_n$: $Q(pp_1, \ldots, pp_n)$ iff $P(t_1, \ldots, t_n)$ for the bearers $pp_1, \ldots, pp_n$ of $t_1, \ldots, t_n$.

b. If f is an n-place function on number tropes, then for some n-place function on pluralities g, for any number tropes $t_1, \ldots, t_n$: $g(pp_1, \ldots, pp_n) = f(t_1, \ldots, t_n)$ for the bearers $pp_1, \ldots, pp_n$ of $t_1, \ldots, t_n$.

Again, $pp_1, pp_2, \ldots$ are plural variables standing for several objects at once.

What about the predicates *natural*, *rational*, and *real*? These are technical predicates that already at the outset are defined just for the domain of all numbers, rather than only the natural numbers. They will therefore not be applicable to number tropes, which are outside the domain of their application.

The possibility of some mathematical properties and functions being applicable to number tropes on the basis of operations on concrete pluralities is also reflected in the acceptability of descriptions of agent-related mathematical operations on number tropes:

(28) a. John added the number of children to the number of adults, and found there were too many people to fit into the bus.

b. John subtracted the number of children from the number of invited guests.

Addition as a mathematical operation performed by an agent, as in (28a), is possible with number tropes for the same reason as addition as a mathematical function. What matters is that the operation as an operation on number tropes is definable in terms of an operation on the underlying pluralities. This does not necessarily mean that when John added the number of children to the number of adults, he first mentally put together the plurality of children with the plurality of adults and then counted the result. It just means that if he obtained the correct result, he might just as well have obtained it by performing an operation on the concrete pluralities first.

Subtraction of a number trope t from a number trope t' is possible just in case the plurality that is the bearer of t' includes the plurality that is the bearer of t. Thus speakers do not generally accept (29a):

(29) a. ?? John subtracted the number of planets from the number of invited guests.

There is an available reading, though, of (29a), a reading more naturally available in a case like (29b):

(29) b. John subtracted the number of passports from the number of applicants.

The reason why (29b) is possible is obviously that it presupposes a natural 1–1 association between passports and applicants. Subtraction will then be an operation on pluralities as well: start with the applicants, associate them with their passports and take away the passports together with their associated applicants, and the number of the remaining applicants will the result of the subtraction.

Division of one number trope by another is also not easily available. Thus speakers do not generally accept (30a). Though when the second term is a numeral, as in (30b) it is generally judged unproblematic, not so, however, when the first term is a numeral and the second a number trope term, as in (30c):

(30) a. ?? John divided the number of invited guests by the number of planets.
b. John divided the number of invited guests by two.
c. ?? John divided eighteen by the number of invited guests.

Divide by two is a complex predicate that involves an arithmetical operation definable as an operation on a plurality. By contrast *divide eighteen* by is not such a predicate: eighteen is not associated with a particular plurality that a division could target, and the plurality of a number trope is not something by which it could be divided.

Again, as with subtraction, there are circumstances, under which a sentence like (30a) is acceptable, for example in the circumstances of (31):

(31) John divided the number of invited guests by the number of tables.

(31) is possible, obviously, because there is a concrete point in associating guests with tables. John's mathematical operation in (31) naturally goes along with an operation on the underlying pluralities, namely an association of each table with different guests, so that if possible the same number of guests is assigned to each table (that is, the guests of a given table can be mapped 1–1 onto the guests of another table). Thus, again, division is possible because it corresponds naturally to an operation on concrete pluralities.

Multiplication with number tropes also is available in certain circumstances:

(32) a. John doubled the number of invited guests.
b. Three times the number of children can fit into the bus.

Those examples, crucially, involve number tropes both as a point of departure and as the result of the multiplication. In (32a), John's act of 'doubling' consists not just in a mathematical operation, but in the replacement of one number trope (the number of invited guests at time t) by another (the number of invited guests at t'). In (32a), the doubling of the number trope may consist in adding as many names as there already are on the list of invited guests. Also (32b) does not just describe a mathematical operation of multiplication of the number of children by three, but rather compares the actual number of children to a hypothetical number trope whose bearer consists in a maximal number of children that fit into the bus. (32b), that is, compares the actual number of children to a hypothetical number trope with three times as many children as bearers.

Arithmetical operations thus are possible with number tropes just in case they can be defined as operations (of a simpler or a more complicated sort) on the underlying pluralities. It is then expected that 'mixed operations' involving both number tropes and pure numbers are excluded. This is indeed the case:

(33) a. ?? John subtracted the number ten from the number of children.
  b. ?? John added the number twenty to the number of children.

Number tropes can have only those mathematical properties that are derivative of operations on the underlying pluralities. In addition, number tropes have empirical properties tied to the particular nature of their bearers, properties pure numbers do not have. The difference in the range of properties number tropes and pure numbers may have also shows in the way general property-related expressions are understood with number trope terms and explicit number-referring terms. Such expressions include *investigate*, *property*, and *behavior*:

(34) a. John investigated the number 888.
  b. John investigated the number of women.

(35) a. the properties / behavior of the number 8
  b. the properties / behavior of the number of women

Whereas (34a) can only mean that John investigated the mathematical properties of 888, (34b) implies that John's investigation was also an empirical one regarding the women in question, namely how many women there were. Similarly, whereas (35a) can only refer to the mathematical properties or the mathematical behavior of a number, (35b) also refers to non-mathematical, empirical properties or behavior of the plurality of women.

6.5 APPARENT IDENTITY STATEMENTS

Let us now turn to the problem of apparent identity statements like (1), repeated below:

(1) The number of planets is eight.

One sort of evidence that (1) is not an identity statement involving two number-referring terms comes from the semantic unacceptability of the sentences below:

(36) a. ?? The number of planets is the number eight.

b. ?? Which number is the number of planets?
c. ?? The number of planets is the same number as eight.

But there is even more conclusive evidence that (1) is not an identity statement, to which I will come shortly.

One obvious alternative analysis of (1) to that as an identity statement is an analysis as a subject-predicate sentence, with the subject referring to a trope and the numeral acting as a predicate of tropes. But this cannot be right. First of all, as was said already, a trope does not 'have' the property it instantiates, that is, a trope instantiating the property of being eight is not 'eight' itself. Moreover, the proposal cannot be right for syntactic reasons: subject-predicate sentences generally do not allow for inversion, as seen in (37) (Heycock and Kroch, 1999), whereas (1) does, as seen in (1′):

(37) a. John is honest.
b. *Honest is John.

(1′) Eight is the number of planets.

There is a third kind of sentence besides identity statements and subject-predicate sentences for which (1) is a candidate and that is a specificational or pseudocleft sentence (Higgins, 1973; Heycock and Kroch, 1999). A specificational sentence typically involves a wh question or question-like expression in subject position and a not necessarily referential expression in postcopula position. A typical example is (38a), where the subject takes the form of an indirect question and the postcopula expression is a verb phrase, which is a non-referential expression:

(38) a. What John did is kiss Mary.

One important analysis of specificational sentences takes them to express relations between questions and answers (den Dikken et al., 2000; Schlenker, 2003; Romero, 2005).[10] The answer may of course consist in the content of a nonreferential expression, with a complete answer being a completion of that expression as a full sentence.

Crucially, specificational sentences allow for inversion:

(38) b. Kiss Mary is what John did.

(38a) illustrates the most important type of a specificational sentence, in which the subject is a wh-phrase and thus arguably an indirect question. However, there are also specificational sentences with a definite NP as subject, such as:

(39) The biggest problem is John.

Here the subject would be a 'concealed question', a non-interrogative expression whose meaning, though, is question-like (Grimshaw, 1997). In (39), *the biggest problem* will then stand for a question of the sort 'what is the biggest problem?'.

[10] An alternative analysis takes specificational sentences to express higher-order equations, see Jacobson (1994).

There is a particularly strong piece of evidence that (1) is in fact a specificational sentence, rather than an identity statement. It comes from the choice of pronouns in the subject position of specificational sentences in German.

English specificational sentences may contain the pronoun *that* or *it* in subject position, pronouns that can be anaphoric to a preceding concealed question (Mikkelsen, 2004):

(40) a. The biggest problem is John; it is not Bill.
b. What is the biggest problem? That certainly is John.

In English *it* and *that* as in (40a, b) can also be used as ordinary pronouns referring to objects. By contrast, German pronouns in the subject position of specificational sentences can only be *das*, 'that' or *es* 'it', not pronouns inflected for gender, such as *sie* 'she'. German *die Zahl der Planeten* 'the number of planets' is feminine, but the only pronoun that can replace it is *es* (neutral) or (more colloquial) *das* as in (41a), unlike in ordinary identity statements as in (41c), where the feminine pronoun *sie* would have to appear:

(41) a. Die Zahl der Planeten ist acht. Früher dachte man, es wären neun.
'The number of planets is eight. Before it was thought that it was (pl) nine'.
b. ?? Die Zahl der Planeten ist acht. Früher dachte man, sie wäre neun.
'The number of planets (fem) is eight. Before it was thought that she was nine.'
c. Maria ist nicht Susanne, sie / *es ist Anna.
'Mary is not Sue, she / *it is Ann.'

The German data indicate that (1) (and not just its German correlate) is in fact a specificational sentence, with its subject being a concealed question. That is, *the number of planets* in (1) will have as its denotation a question or question-like entity of the sort 'how many planets are there?'.

CONCLUSION

For Frege, the construction *the number of planets* was not only indicative of the ontological status of numbers as objects. It was also revealing as to the nature of numbers themselves, namely as objects obtained by abstraction from concepts (Hume's Principle). In this paper, we have seen that *the number of*-terms are not number-referring terms and moreover are not obtained by a functor applying to a concept-denoting term. Of course, this does not show that Fregean or Neo-Fregean conception of numbers as objects is mistaken as such, but it means that there is no support for it from natural language.

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