Workshop *The Semantics of Cardinals*

Columbus, Ohio, March 5, 2014

**Number Terms as Proper Names**

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**1. The Fregean / Neo-Fregean (Hale, Wright) view**

Syntactic criterion for objecthood:

An object is what can be referred to by a referential term (a ‘name’).

Referential terms have the semantic function in a sentence to stand for an object.

Criteria for referential terms (also Neo-Fregean view (Hale, Wright):

* can flank both sides of the identity symbol
* occur with definite determiner
* Hale (1987) (roughly): referential terms can be replaced by pronouns playing the role of a bound variable

Putative cases of referential terms referring to numbers:

[1] *the number eight*

[2] *eight,*  as in *eight is divisible by two*

[3] *the number of planets*

The view of Moltmann (2013a, b):

*- The number eight* is a reifying term; it refers to an abstract object introduced on the basis of the number word *eight* (or the plural concept ‘eight’)

Other ‘reifying terms’: *the concept horse, the truth value true*

*- Eight*: not referential, but may appear in referential position, retaining its meaning as a plural predicate which it has when occurring as an adjective.

- *The number of planets*: referential term, but does not refer to a number as an abstract object, but a number trope: the planets reduced to just one aspect: how many they are, or the manifestation of the property of being eight in the planets

The motivations for the view

*The number of*-terms permit predicates of concreteness (perception, causation, evaluation).

*The number of*-terms allow for only rudimentary arithmetical operations, just those operations that can be understood as operations on number tropes:

(1) a. John added the number of men and the number of women.

 b. ??? the successor of the number of planets.

*The number of* can also act as a numeral replacement, as below:

(1) c. John invited a great number of people.

Frege (1884) about (2):

(2) The number of planets is eight.

*The number of planets* is a referential term. (2) expresses an identity statement.

*Eight* and *the number of the planets* are number-referring terms.

Frege: (2) is equivalent to:

(2’) a. ?? The number of the planets is the number eight.

 b. ?? Die Anzahl der Planeten ist die Acht.

 ‘The number of the planets is the eight.’

The analysis of Moltmann (2013a, b):

The sentence is a specificational sentence just like:

(3) a. What John did was kiss Mary.

 b. The date of the exam is tomorrow.

*The number of* has the numeral replacement function,

*The number of planets* stands for the question ‘how many planets are there?’

or better ‘what number of planets are there?’

*Eight* stands for a plural property, constituting an elliptical answer.

Moltmann (2013a, b):

Specific evidence (from German) for the nonreferential status of simple number terms (*eight*).

In this talk:

This evidence is mistaken: simple number terms in German are names that are ‘number-neutral’ or ‘mass’, which explains the apparent evidence for nonreferentiality.

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**2. Simple number words as referential terms**

**2.1. Mathematical and nonmathematical properties**

(4) a. Four is divisible by two.

 b. The number four is divisible by the number two.

(5) a. John added two to four.

 b. John added the number two to the number four.

(6) a. Twelve, which is divisible by two, is not a prime number.

 b. Twelve, which is smaller than fifteen, is greater than ten.

(7) a. The number twelve, which is interests me a lot, is an important number in religious

 and cultural contexts.

 b. ?? Twelve, which interests me a lot, is an important number in religious and cultural

 contexts.

(8) a. the number twelve, which I like to write my dissertation about, …

 b. ?? twelve, which I like to write my dissertation about, …

*eight*: only mathematical properties

*the number eight*: one-place mathematical properties and non-mathematical properties

The sortal *numbe*r in the predicate introduces the same objects as in an NP:

(9) a. Twelve is a number I like to write my dissertation about.

 b. # I like to write my dissertation about twelve.

Predicates taking only numerals, but not explicit number referring terms:

The predicates *count*:

(10) a. Every day, John has to count the visitors. Today he counted ten, yesterday he counted

 two, before yesterday he counted zero.

 b. ??? John counted the number ten.

Two-place mathematical operations:

(11) a. Two and four are / is six.

 b. ??? The number two and the four is the number six.

**2.2. Accounting for the difference between simple number words and explicit number-referring terms**

Hofweber’s (2005) cognitive type shift:

Simple number words retain semantic type of determiners, but have syntactic status of singular terms.

Refinement: With predicates expressing mathematical properties simple numerals retain semantic type of adjectives, but act syntactically as singular terms. Simple numerals are not interpretable with predicates expressing nonmathematical properties.

The *number* eight:

Sortal number ensures reification of numbers as objects. Object so introduced will obtain all the properties determined by the semantic context of the semantic value of the corresponding numeral, but in addition can bear nonmathematical properties.

Compositional semantics:

*number*: takes numeral, as nonreferential term, maps plural property to reified number.

Reifying terms involve quotation:

(12) a. \* the number that number

 b. \* the number what I wrote down

 c. \* the number what I added to two

**2.3. Special quantifiers**

*Something* in general: can replace nonreferential expressions (predicates, *that*-clauses, plurals)

Can replace numerals:

(13) a. John added something to something else, namely he added ten to twenty.

 b. John added something to the number of children: he added two.

 c. John counted the same thing as Mary when he counted the visitors, namely

 twenty three.

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**2.4. Explanation of the difference between mathematical and nonmathematical predicates with simple number terms**

The **Adjectival Strategy** in the philosophy of mathematics (Hodes 1984):

(14) a. Two and two is two is four.

 b. If there are (were) two things and two other things, then there would be four things.

 c. ( 2 x Fx &  2 x Gx &  x (Fx & Gx))   4 x (Gx v F x)

The Adjectival Strategy can apply to predicates expressing mathematical properties, but not to predicates expressing nonmathematical properties.

Accounting for the quantifiers *something* replacing simple number words:

*Something* as a general plural quantifier:

(15) a. John ate something, namely the apples.

 b. Something plus something is four.

 c. Some xx (F) plus some xx (G) is four (F, G)

*Something* as a reifying quantifier:

(15) d. John ate several things, the apples, the pears, and the raisins.

**3. Apparent evidence for the nonreferential status of numerals (in German)**

**3.1. Two kinds of relative pronouns in German**

Sortal-independent relative pronouns: w-pronouns:

*alles, was; nichts, was, vieles, was*

‘everything that’ , ‘nothing that’, ‘many things that’

Sortal-dependent relative pronouns : d-pronouns *das, der, die, die*

*Der Mann der ‘the man who’, die Frau, die* ‘the woman who’*, das Kind, das ‘*the child that’, *das Ding, das / alle Dinge, die* ‘the thing that / all things that’

Nonreferential NP complements

*That*-clauses:

(16) a. Dass S, was / \* das mir schon seit langem bewusst war, ist allen bekannt.

 ‘That S, which I have been aware of for a long time, is known to everyone.’

 b. Die Tatsache dass S, die / \* was mir schon seit langem bewusst was, ist allen bekannt.

 ‘The fact that p which I have been aware of for a long time, is known to everyone.’

 c. Hans hofft, dass die Sonne scheinen wird, was / \* das ich nicht hoffe.

 ‘John hopes that the sun will shine, which I hope too.’

Predicative NPs:

(17) Hans wurde Musiker, was / \* das Maria auch wurde.

 ‘John became a musician, which Mary became too.’

Intensional NPs:

(18) Hans such eine Sekretaerin, was / \* die Maria auch sucht.

 ‘John is looking for a secretary, which Mary is looking for too.’

Quotation:

(19) Er nannte sie Maria, was / \* das er sie noch nie genannt hatte.

 ‘He called her Marie, which he had never called her before.’

Generalization (Moltmann 2013a, b): W- pronouns are an indication of non-referentiality.

Simple number terms take w-pronouns, not d-pronouns:

 (20) a. zwoelf, was / \* das / \*? die eine Zahl ist, die mich sehr interessiert, …

 ‘twelve, which is a number that interests me a lot, …’

 b. zwoelf, was / \* das / \* die durch zwei teilbar ist, …

 ‘twelve, which divisible by two, …’

 c. die Zahl zwoelf, die / \* was durch zwei teilbar ist, …

 ‘the number twelve, which divisible by two, …’

**3.2. Lack of support of plural anaphora**

Nonreferential complements

(21) a. Hans fuerchtet, dass es regnen wird und dass kaum jemand kommt. Maria fuerchtet

 das / \* sie auch.

 ‘John fears that it will rain and that hardly anyone will show up. Mary fears that /

 \* them too.

 b. Maria wurde eine gute Geigenspielerin und eine ausgezeichnete Kuenstlerin. Anna

 wurde das / \* sie auch.

 ‘Mary became a good violinist and an excellent artist. Mary became that / \* them too.’

 c. Hans braucht eine Sekretaerin und einen Assistenten. Maria braucht das / \* sie auch.

 ‘John needs a secretary and an assistant. Mary needs that / \* them too.’

 d. ‘Anna’ und ‘Marie’ sind zweisilbig. ??? Sie sind nicht dreisilbig.

 ‘’Anne’ and ‘Marie’ are dysillabic. They are not trisyllabic.’

Simple number words in German:

(22) a. Hans addierte zehn und zwanzig.

 ‘John added ten and twenty. ‘

 b. ??? Maria addierte sie auch.

 ‘Mary added them too.’

 c. Maria addierte diese Zahlen auch.

 Mary added those numbers too.

(23) a. Hans addierte die Zahlen zehn und zwanzig.

 ‘John added the numbers ten and twenty.’

 b. Maria addierte sie auch.

 ‘Mary added them too.’

(24) a. Hans notierte zehn und zwanzig.

 ‘ John wrote down ten and twenty. ‘

 b. ?? Maria notierte sie auch.

 ‘Mary wrote them down too.’

 c. Maria notierte diese Zahlen auch.

 ‘Mary wrote those numbers down too.’

(25) a. Hans notierte die Zahlen zehn und zwanzig.

 ‘ John wrote down the numbers ten and twenty.’

 b. Maria notierte sie auch.

 ‘Mary wrote them down too.’

(26) Drei und fuenf sind ncht durch zwei teilbar. ?? Sie sind / ok Das sind beides Primzahlen.

 ‘Three and five are not divisible by two. They are both prime numbers.’

**3.3. Verbs and the mass-count distinction**

Verbs classify as mass rather than count, regardless of their actionsart (Moltmann 1997):

(27) John greeted Mary and kissed Sue. He did that / \* them this morning.

(28) a. John woke up three times.

 b. \* John woke up three.

(29) John woke up frequently.

(30) Hans sang, was / \* das Maria auch tat.

 ‘John sang, which Mary did too.’

**4. The problem**

**4.1. Names for locations in German**

Choice of relative pronouns:

(31) a. Deutschland, was / # das ich gut kenne

 ‘Germany, which I know well’

 b. Berlin, was / # das mir besser gefaellt als Muenchen

 ‘Berlin which pleases me more than Munich,

(32) die Stadt Berlin, die / # was mir gut gefaellt

 the city Berlin, which pleases me

Plural anaphora support

Proper names:

(33) Hans mag Susanne und Maria. Bill mag sie auch.

 ‘John likes Susanne and Mary. Bill likes them too.’

Simple names for locations:

(34) a. Ich habe Berlin und Muenchen besucht. Maria hat ??? sie / diese Staedte auch besucht.

 ‘I have visited Berlin and Munich. Mary has visited them / those cities too.’

 b. Ich mag Frankreich und Italien. Marie mag ?? sie / ok diese Laender auch

 ‘I like France and Italy. Mary likes them too.’

(35) Ich habe die Stadt Berlin und die Stadt Muenchen besucht. Maria hat sie auch besucht.

 ‘I know the city of Berlin and the city of Munich. Mary has visited them too.’

Contrast to English:

(36) a. I know Berlin and Munich. Mary knows them too.

 b. I like France and Italy. Mary likes them too.

 c. I would like to visit Australia and Africa. Mary would like to visit them too.

**4.3. Names for times**

(37) John remembers 1960 and 1963. Mar remembers # them / ok those years too.

(38) a. 1960, was / \* das interessanter ist als 1970

 ‘1960, which is more interesting than 1970’

 b. Montag, was / \* der mir besser passt als Dienstag, ist ein Feiertag.

 ‘Monday, which suits me better than Tuesday is a holyday.’

(39) das Jahr 1960, das / \* was interessanter ist als 1970, …

 ‘the year 1960, which is more interesting than 1970, …’

(40) a. Ich habe an 1960 und 1970 gedacht. Maria hat auch an \* sie / ok diese Jahre gedacht.

 ‘I have thought about 1960 and 1970. Mary thought about \* them / ok those years too.’

 b. Anna schlug Montag und Dienstag vor. Maria schlug \* sie /ok diese Tage auch vor.

 ‘Ann proposed Monday and Tuesday. Mary proposed \* them / those days too.’

**4.4. Mass NPs used as kind terms**

(41) a. Magnesium is an important mineral.

 b. White gold is rare.

(42) a. Magnesium, was / ??? das lebenswichtig is

 ‘magnesium, which is of vital importance’

 b. Wasser, was / ??? das gesuender ist als Bier

 ‘water, which is healthier than beer’

(43) a. Gold und Silber werden zum Schmuckherstellen verwendet. \* Sie glaenzen.

 ‘Gold and silver are used to make jewelry. They are shiny.’

 b. Magnesium und Eisen sind lebenswichtig. Jeder braucht \* sie / ok das (beides).

 ‘John needs magnesium and iron. Mary needs them / that / that both.’

**4.5. New generalization and consequences for simple number terms**

W-relative pronouns in German go with sortal-free NPs. Sortal-free names classify as ‘mass’ or ‘number neutral’. Proper names in German and English have an implicit sortal content ‘person’. Names for locations, times, kinds in German have no sortal content (but they do so in English).

Mass or number-neutral conjoined NPs do not support plural anaphora.

(44) John drank water and wine / the water and the wine. Mary drank ??? them / ok it two.

**Consequences for the mass-count distinction:**

The presence of a sortal classifier / sortal concept / singular count or plural morphology is crucial for a category to classify as count, individuation or conceptual content is not enough.

**Consequences for simple number terms**

Simple number terms in German are sortal free, hence number-neutral or mass.

Simple number terms in English involve a sortal concept ‘number’ as part of their lexical content. But this appears incompatible with them retaining the meaning of their adjectival or determiner use (Hofweber 2005, Moltmann 20013a, b). Simple number terms rather acts as names referring to what falls under the sortal concept associated with them.

Accounting for the semantic differences between *eight* and *the number eight* :

Acceptability with *eigh*t, but not *the number eight* with *count*:

*Count* is an intensional verb, goes with indefinite bare adjective *eight*, but not definite / referential the number eight.

*Two and four is six*: Such sentences involve a special meaning of *and* and *is*, taking bare adjectives only, not referential NPs like *the number eight*.

**5. Close appositions and definite number terms in German**

Number terms with definite determiners in German

(45) a. die acht, die / ??? was keine Primzahl ist

 ‘the eight, which is not a prime number’

 b. Hans addierte die zwei und die vier. Marie addierte sie auch.

 ‘John added the two and the four. Mary added them too.’

Definite determiners in German names:

(46)? Die Maria kam. (dialectal)

 ‘The Mary arrived’.

Close appositions

(47) the poet Goethe

Names for mountains, lakes and temples (in German)

(48) a. der Vesuv, der b. der Etna, der

 c. der Parthenon, der d. der Houriaji, der

(49) Wir haben \* Parthenon / ok den Parthenon / ok den Parthenontempel besichtigt.

 ‘We have visited Parthenon / the Parthenon / the Parthenon temple.’

*Die acht* =  *die Zahl acht* ?

But constraints:

(50) die eins, …. die acht, … ? die zwanzig, … ??? die dreihundert

 ‘the one … the eight, .., the twenty, …, the threehundred’

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