NPs with intensional relative clauses such as *the book John needs to write* pose a significant challenge for semantic theory. Such NPs act like referential terms, yet they do not stand for a particular actual object. This paper will develop a semantic analysis of such NPs on the basis of the notion of a variable object. The analysis avoids a range of difficulties that a more standard analysis based on the notion of an individual concept would face. Most importantly, unlike the latter, the proposed analysis can be carried over NPs such as *the number of people that fit into the bus*, which describe tropes (particularized properties).

1 Introduction

Individual concepts, functions from possible worlds and times to entities, have become a standard tool in linguistic semantics since Montague (1973), in particular as semantic values of functional NPs such as *the president of the US* in the context below:

(1) The president of the US is elected every four years.

If making use of individual concepts, *the president of the US* will stand for a function mapping a world \( w \) and time \( t \) to the individual that is the president of the US in \( w \) at \( t \).

This paper focuses on NPs with relative clauses containing an intensional verb, as in (2), for which an analysis in terms of individual concepts is equally tempting:

(2) The book John needs to write must have more impact than the one he has already written.

I will call NPs of this sort, that is, definite NPs modified by a relative clause with an intensional verb as predicate ‘IR-NPs’. On an analysis using individual concepts, *the book John needs to write* would stand for a function mapping a circumstance in which John fulfills his need to a book written by John in that circumstance.
In fact, there is a construction closely related to the one in (2) for which an analysis in terms of individual concepts has been proposed. This is the construction below, analysed by Grosu & Krifka (2007), which involves an intensional verb and the copula verb be:

(3) The gifted mathematician John claims to be could solve this problem in no time.

On Grosu & Krifka’s (2007) analysis, roughly, the subject of (3) stands for an individual concept mapping any circumstance in which what John claims is true onto a gifted mathematician identical with John in that circumstance.

I will argue that an analysis based on individual concepts raises a range of problems, ontologically, conceptually, and empirically. Focusing on the construction in (2), I will make use of an alternative to the notion of an individual concept, namely the notion of a variable object. A variable object is an entity that may have different manifestations as different individuals in different actual or counterfactual circumstances. The notion of a variable object derives from the notion of a variable embodiment, which plays a central role in Kit Fine’s metaphysics (Fine 1999). Unlike individual concepts, variable objects are entities, which means, given standard type theory, they are of type $e$. Individual concepts are not so much entities of a particular type, but rather they make up the contribution of certain sorts of occurrences of expressions to the compositional meaning of the sentence. Their type-theoretic type $\langle s, t \rangle$, the type of functions from indices to individuals, captures the fact that NPs of that type can occur only in syntactic contexts in which such a function can compose suitably with the semantic value of a sister constituent. Variable objects, by contrast, are objects and as such able to act as semantic values of referential NPs, which will include functional and IR-NPs.

The main motivation for positing variable objects as the semantic values of functional and IR-NPs comes from constructions in which those NPs appear to describe the bearers of tropes. Tropes are particularized properties, concrete manifestations of properties in objects. Typical examples of tropes are ‘the softness of the pillow’ and ‘the redness of the apple’, which are qualitative tropes. ‘The length of the paper’ and ‘the number of planets’ can also be considered tropes, namely quantitative tropes. ‘The number of planets’ will be a number trope, the instantiation of the property of being eight in the plurality of planets (Moltmann 2013a,c). In the sentences below, IR-NPs appear to describe the bearers of tropes: number tropes in (4a) and tropes of causal effect in (4b):

(4) a. The number of people that fit into the bus exceeds the number of people that fit into the car.
b. The impact of the book John needs to write must be greater than the impact of the book he has already written.

There is a crucial difference between (4a) and (4b), though. (4b) requires a modal in the main clause, which (4a) does not. (4b), without the modal, would be unacceptable. As will be discussed, this means that only the NPs in (4a) refer to tropes with variable objects as their bearer. The NPs in (4b), by contrast, refer to ‘variable tropes’, variable objects whose manifestations are tropes rather than individuals.

The variable-objects account of IR-NPs goes along with a particular compositional analysis of the construction, according to which the head of the relative clause is interpreted in the lower position inside the relative clause. Thus, the account presupposes a syntactic view according which the head noun of an IR-NP originates from inside the relative clause, within a copying theory of movement or a view according to which it can be reconstructed into the lower position.

The paper starts with a few clarifying remarks about tropes and reference to tropes in natural language. It then discusses in greater detail the individual-concept approach to the constructions in (1), (2), and (3). The main part of the paper consists in the introduction of the notion of a variable object, which will first be applied to the semantics of (1) and (2) and then to the semantics of (3) as well. The ontology of variable objects will also be used to explain when IR-NPs require a modal in the main clause and when they don’t.

2 Reference to tropes in natural language

It is a common view, at least since Aristotle’s ‘Categories’, that terms of the sort in (5) refer to tropes or particularized properties, that is, particular, non-sharable features of individuals (Williams 1953, Strawson 1959, Wolterstorff 1970, Campbell 1990, Lowe 2006, Mertz 1996):

(5)  a. the wisdom of Socrates  
    b. the softness of the pillow  
    c. the simplicity of the dress

According to that view, (5a) refers to the particular manifestation of wisdom in Socrates, that is, a wisdom trope that has Socrates as its bearer, (5b) to the manifestation of softness in the pillow, and (5c) to the manifestation of simplicity in the dress.

There are equally good reasons to take the terms below to refer to tropes, namely quantitative tropes, manifestations of being so and so tall or so and so long in an individual (Campbell 1990, Moltmann 2009, 2013a):
(6)  a. the height of the building
    b. the length of the paper

Quantitative tropes also include number tropes, such as the manifestation of the
property of being eight in the plurality of the planets, as below (Moltmann 2013a,c):

(6)  c. the number of planets

The number trope that is the manifestation of the property of being eight in the
plurality of the planets is a particularized property not shared by any equally
numbered plurality.

Qualitative and quantitative tropes, as we refer to them in natural language,
exhibit the very same properties characteristic of tropes, which I will turn to now.

2.1 The relation of a trope to its bearer

An important feature of tropes is their dependence on a bearer. Socrates is the
bearer of the trope ‘Socrates’ wisdom’ and the planets are the bearers of the trope
‘the number of planets’. A trope exists in a world \( w \) at a time \( t \) only if its bearer
exists in \( w \) at \( t \). Moreover, two tropes are identical only if their bearers are identical.

2.2 Similarity relations

Another important feature of tropes consists in the way they enter similarity rela-
tions. Tropes instantiating the same property are similar, and tropes instantiating
the same ‘natural’ property are exactly similar. For example, two redness tropes
are similar, and two tropes exhibiting the very same shade of red are exactly sim-
ilar. In natural language, exact similarity is expressed by *is the same as* (which
does not imply numerical distinctness), as below:

(7)  a. The quality of this fabric is the same as the quality of that fabric.
    b. The impact of John’s book was the same as the impact of Bill’s book.
    c. The height of the desk is the same as the height of the lamp.
    d. The length of John’s vacation is the same as the length of Mary’s vacation.
    e. The number of women is the same as the number of men.

Only the *is* of identity expresses numerical identity, rendering the sentences below
intuitively false:
(8)  a. ?? The quality of this fabric is the quality of that fabric.
    b. ?? The impact of John’s book was the impact of Bill’s book.
    c. ?? The number of women is the number of men.
    d. ?? The height of the desk is the height of the lamp.

The way is the same as and the is of identity are understood with the terms in question is a particularly good indication that those terms refer to tropes and not abstract objects such as properties, degrees, or numbers.

2.3 Properties of concreteness

Tropes are as concrete as their bearers. If they have a concrete bearer, they may exhibit properties of concreteness such as the ability of acting as objects of perception – in fact as the immediate objects of perception (Williams 1953, Campbell 1990, Lowe 2006):

(9)  a. John noticed the simplicity of the dress.
    b. John observed Mary’s politeness.
    c. John noticed the small number of women that were present.

Tropes may also act as relata of causal relations (Williams 1953), as in the sentences below:

(10)  a. The heaviness of the bag made Mary exhausted.
    b. The number of passengers caused the boat to sink.
    c. The weight of the lamp caused the table to break.

Tropes furthermore may have a temporal duration:

(11) John’s happiness lasted only a year.

There are other properties of concreteness that tropes may exhibit. One such property is what one may call ‘description independence’. Description independence

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1 More problematic is the spatial location of tropes. Even though the philosophical literature considers tropes to be located in space just where the bearer is located at the relevant time, in fact trope-referring terms in general resist predicates of spatial location:

(i)  a. ?? John’s happiness was in Munich.
    b. ?? John’s heaviness on the table
consists in that tropes generally have an internal structure ‘below’ the description used to refer to them. A manifestation of that property is the applicability of predicates of description and qualitative comparison to tropes:

(12) a. John described Mary’s beauty.
   b. John compared Mary’s beauty to Sue’s beauty.

Tropes differ in that respect from states and facts which strictly match the content of a canonical description and thus do not accept predicates of description and comparison (on a natural reading):

(13) a. ?? John described (the state of) Mary’s being beautiful.
   b. ?? John compared (the state of) Mary’s being beautiful to (the state of) Sue’s being beautiful.

Related to description independence is the ability of tropes to have a measureable extent, allowing, for example, for the application of the predicate exceed, which, again, is not applicable, on a natural reading, to states and facts:

(14) a. Mary’s happiness exceeds Bill’s.
   b. ?? The fact that Mary likes Bill exceeds the fact that Mary is tall.
   c. ?? The state of Mary’s liking Bill exceeds the state of Mary’s being tall.

Tropes referred to with the help of predicates, however determinable, unspecific, or quantificational the predicates may be, are always maximally specific – in contrast to states and facts, entities whose nature is ‘exhausted’ by the content of a canonical description (Moltmann 2013b).

Another characteristic of tropes is that they share with their bearers properties of quantitative comparison. These are properties expressed by predicates like exceed, equal or high. Such predicates apply to tropes as well as their bearers, though in the latter case they require qualification of respect:

(15) a. The eagerness of John exceeds the eagerness of Mary.
   b. The sloppiness of John equals the sloppiness of Mary.

   b. John equals Mary in sloppiness.

(17) a. The height of John exceeds / equals the height of Mary.
   b. John exceeds / equals Mary in height.

(18) a. The number of men exceeds the number of women.
   b. The men exceed the women in number.
(19) a. The number of participants is high.
   b. The participants are high in number.

Those predicates do not apply to abstract objects such as properties or numbers:

(20) a. ?? The property of being very eager exceeds the property of being not so eager.
   b. ?? The property of being extremely sloppy exceeds the property of being somewhat sloppy.
   c. ?? The number 13 does not equal the number 8.
   d. ?? 100 is high.

Thus, predicates of quantitative comparison provide another good indication for trope reference as opposed to reference to abstract objects (Moltmann 2013a,b,c).

3 Trope-reference with intensional relative clauses

We can now try to apply the same types of predicates to NPs as in (4a, b), that is, NPs that appear to refer to tropes, but tropes whose bearers are described by IR-NPs. In general, the various types of predicates can apply, which means that those NPs must indeed refer to tropes.

First, predicates of perception and causation may apply to such NPs:

(21) a. John noticed the number of screws that are missing.
   b. The number of screws that are missing caused the table to fall apart.
   c. Mary was astonished by the length of the paper John needs to write.
   d. Mary noticed the amount of repair that is required to make the machine work again.

Even in the absence of an actual bearer, tropes, under particular circumstances as in (21a, d), may act as objects of perception.

Also predicates of similarity and identity apply to the NPs in question in the same way they did to ordinary trope-referring NPs.

(22) a. The number of women in the room is the same as the number of men in the room.
   b. ??? The number of women in the room is the number of men in the room.
(23) a. The number of books Mary wants to write is the same as the number of books Sue wants to write.
   b. The number of books Mary wants to write is the number of books Sue wants to write.

Furthermore, predicates of quantitative comparison and evaluation are applicable just as they were to ordinary trope-referring terms. Moreover, such predicates can alternatively apply to the description of the bearer of the tropes in the presence of a qualification of respect:

(24) a. The originality of the book John wants to write needs to exceed the originality of the book John has already written.
   b. The book John wants to write needs to exceed the book John has already written in quality.

(25) a. The elegance of the dress that the bridesmaid will wear should not exceed the elegance of the dress that the bride will wear.
   b. The dress that the bridesmaid will wear should not exceed the dress that the bride will wear in elegance.

(26) a. The height of the desk John needs exceeds the height of the desk John is using right now.
   b. The desk John needs exceeds the desk John is using right now in height.

(27) a. John compared the number of books Mary wants to write to the number of books Sue wants to write.
   b. John compared the books Mary wants to write to the number of books Sue wants to write in number.

(28) a. The number of people that fit into the bus is high.
   b. The people that fit into the bus are high in number.

Given the behavior of predicates, the parallel between ordinary trope-referring terms and corresponding terms with intensional relative clauses is rather striking and supports the view that the latter are trope-referring terms as well.

4 Individual-concepts

Before turning to trope-referring NPs with IR-NPs, let us first discuss in more detail the individual-concepts approach to functional and IR-NPs.
It is wellknown that functional NPs act referentially in that they can be antecedents of anaphoric pronouns in subsequent sentences, as can IR-NPs:

(30) a. The president of the US is elected every four years. He occupies the most powerful office.

b. The book John needs to write must be at least 200 pages long. It needs to be finished by next year.

According to the standard Montagovian account, functional NPs stand for individual concepts (Montague 1973). This means they are of a different type than that of referential NPs, namely of type ⟨s, e⟩, rather than of type e, the type of referential NPs. When predicates like change, rise, increase apply to individual concepts, they will also be of a different type than when they apply to individuals. Thus, increase when applying to an individual concept is of type ⟨⟨s, e⟩, t⟩, rather than of type ⟨e, t⟩.

Predicates according to the Montagovian view can in general be lifted to predicates of individual concepts. A predicate P that unlike increase, rise, and change does not have a meaning that specifically takes into account the individual concept can be lifted to a predicate P′ of individual concepts in virtue of a general meaning postulate, as below:

\[ P_{w,t}(f) = 1 \text{ iff } P_{w,t}(f(w,t)) = 1 \text{ for any world } w \text{ and time } t. \]

There is an apparent problem for the individual-concept account of functional NPs, and that is the impossibility of replacing a functional NP as in (29b) by an explicit function-referring NP, as in the intuitively invalid inference below:

(32) John changed his trainer.

John changed a function.

However, the inference does not follow given that the use of individual-concepts needs to be understood within the context of type theory. In the premise of (32), change is of type ⟨e, ⟨⟨s, e⟩, t⟩⟩, whereas in the conclusion it is type ⟨e, ⟨e, t⟩⟩, invalidating the inference.

A more serious problem for the individual-concept approach is the possibility of reference to tropes whose bearers are described by functional NPs. Below functional NPs clearly specify the bearers of tropes:

(29) a. The president of the US is elected every four years.

b. John changed his trainer.

c. The temperature is rising.

d. The number of students has increased.
(33)  a. The decrease of the number of students caused concern.
    b. The increase in the amount of corruption triggered a revolt.
    c. The impact of the increasing number of students is noticeable.
    d. The rise of the temperature caused the drought.

In these examples, functional NPs describe the bearers of single tropes playing causal roles or acting as objects of perception.

When describing the bearers of tropes, functional NPs can hardly be considered standing for individual concepts (being of type \( \langle s, t \rangle \)). Moreover, they could not be considered referential NPs referring to functions (in which case they would be of type \( e \)), since they could not be substituted by an explicit function-referring NP:

(34)  a. ? the change of the function
    b. ? the impact of the function
    c. ? the rise of the function
    d. ? the responsibility of the function

Functions can be bearers of tropes, but they are generally bearers of very different sorts of tropes. As mentioned in the introduction, individual concepts are not meant to be the particular entities that referential NPs refer to, but rather they make up the contribution of the syntactic type of functional NPs to the compositional meaning of the sentence. Functional NPs are not of type \( e \), the type of referential NPs, but of type \( \langle s, t \rangle \), the type of individual concepts. But that means that the individual concepts they stand for are not entities of a particular sort at all. Tropes, however, are entities, entities that essentially depend on other entities as their bearers. Such entities must be semantic values of NPs of type \( e \) and cannot be semantic values of functional NPs, being of type \( \langle s, \langle e, t \rangle \rangle \).

IR-NPs present similar problems. First of all, there are object-related predicates that can apply to IR-NPs, such as count, describe and enumerate:

(35)  a. John counted the screws that were missing.
    b. John described the personnel that the company needed to hire.
    c. John enumerated the things that he needed to buy.

Such predicates apply in (35) with the very same meaning they have when applying to ordinary objects. They do not obtain a derivative meaning according to which they keep track of the values of a function when applied to different circumstances.\(^2\)

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\(^2\) Note that count applies with the very same meaning to ordinary NPs and IR-NPs:

(i)  a. What did John count?
    b. John counted the legs of the chair and the screws that were missing.

Thus, ordinary NPs and IR-NPs should better not be distinguished in type.
IR-NPs moreover describe the bearers of quantitative tropes in examples such as (4a), repeated below as (36a), and (36b):

(36)  a. The number of people that can fit into the bus exceeds the number of people that can fit into the car.
    b. The length of the vacation John is allowed to take exceeds the length of the vacation Mary is allowed to take.

In (36a) and (36b) the predicate *exceed* applies to two quantitative tropes whose bearers are described by IR-NPs.

Object-related predicates and trope reference thus indicate that IR-NPs are of type $e$, referring to entities of a suitable sort, just like functional NPs. These entities, on the present view, will be variable objects.

The apparent trope-referring NPs below present a different case:

(37)  a. The beauty of the landscape has changed.
    b. The amount of corruption in this administration has become more noticeable.
    c. The impact of this book has diminished.

(38)  a. The number of students has increased.
    b. the increasing number of students

(39)  a. The number of teachers sometimes exceeds the number of students.
    b. The increasing number of students causes problems for the availability of sufficient class rooms.

The NPs in such sentences do not refer to single tropes. Rather (37), (38), and (39) involve typical predicates applying to ‘individual concepts’. On the individual-concepts approach, this means that the subject NPs would stand for functions from circumstances to tropes. On the present approach, it means that they refer to variable tropes.

5 Semantic problems with individual-concepts

There are also problems regarding the compositional semantics of IR-NPs for an analysis using individual concepts. There are two options of analyzing (40) as standing for an individual concept:

(40)  a. the book John needs to write
I will only outline the two options, focusing on the general idea and their problems and leaving out any formal details.

The first option would be an extension of Grosu & Krifka’s (2007) analysis of (41):

(41) the gifted mathematician that John claims to be

Their analysis involves several assumptions. First, it involves type-lifting of all predicates to predicates of individual concepts and all singular terms (including proper names) to terms for individual concepts. Second, it requires treating all intensional verbs as operators quantifying over circumstances (possible worlds and times). Finally, it interprets the head noun *book* in (40a) in the upper position, rather than reconstructing it into the lower position inside the relative clause. Greatly simplifying, this analysis would yield the following as the denotation of (40a):

(40) b. \( \min(\{f \mid \text{book}(f)\} \cap \{f \mid \text{[John needs to write]}(f)\}) \)

The second set mentioned in (40b) would be the set of partial functions mapping a world \( w \) compatible with the satisfaction of John’s needs to an object John writes in \( w \).

This analysis raises a range of problems. First of all, it involves an excessive use of individual concepts, involving a lifting of all predicates and singular terms to the type of individual concepts. While raising singular terms and argument positions of predicates to the type of individual concepts is not as such problematic technically, the move seems too far-reaching given the motivation of just getting the semantics of the construction in (40a) right. The construction in (40a) should not really be grounds for abandoning the view that names stand for individuals and that predicates in natural language are generally predicates of individuals.

Second, the analysis makes a rather problematic philosophical assumption by considering all intensional verbs, including attitude verbs, operators quantifying over circumstances. Certainly a great number of philosophers will find such an assumption problematic. The most common view about attitude verbs is that they are not to be analysed as operators quantifying over circumstances, but rather express two-place relations between agents and propositions. Furthermore there are philosophical views according to which modal verbs do not represent operators quantifying over circumstances but correspond to primitive operators (‘modalism’).\(^3\) The semantic analysis of the construction in (40a) should better not involve a highly controversial philosophical assumption. Rather it should be

\(^3\) See, for example, Forbes (1985).
independent of particular views of attitude verbs, modal verbs, or other intensional verbs.

A third problem for the analysis in (40b) concerns a problem of uniqueness, a problem which specifically arises when carrying over Grosu & Krifka’s (2007) analysis of (41) to the construction in (40a). The problem of uniqueness does not arise for the construction in (41) because Grosu & Krifka (2007) consider the copula be in that construction be the be of identity, which means it takes two individual concepts as arguments.

The uniqueness problems arises for the book John needs to write because in a given world in which John’s need is satisfied, John may have written more than one book meeting his need. In order to guarantee uniqueness, a move is required from the notion of a context of evaluation to that of a truthmaking situation, or rather a satisfaction situation, a situation satisfying John’s need.

A satisfaction situation may also impose various constraints on the book John writes in it (constraints the speaker in fact need not know about): not all the books John writes in a world in which his need is satisfied qualify as ‘the book John’s needs to write’. The complement of need may give only a partial characterization of the exact need. Thus, the individual concept denoted by the book John needs to write should not take into account entire worlds in which John’s need is satisfied, but rather situations satisfying John’s need. More precisely, these situations should not just be situations in which John’s needs are satisfied, but situations wholly relevant for the satisfaction of John’s needs. That is, they should be exact satisfiers of John’s needs, or exact truth makers of the conditions making up John’s needs.4

A given world in which John’s needs are satisfied may contain several situations satisfying his need, each containing a different book.5

The analysis in (40b) crucially relies on the notion of a circumstance of evaluation, but what is needed is the notion of an exact satisfier or an exact truth maker of the condition in question, John’s need. A circumstance of evaluation would not be able to fulfil the latter role because the purpose of the context of evaluation is not only to relativize the truth value of a sentence, but also to fix at the same time the extensions of singular terms and predicates.

In (40a), the truth-making circumstances appear to be determined by a particular condition constitutive of John’s need at a time. However, the truth-making circumstances may also depend on a particular event described by the verb, as in the examples below:

5 For further motivations for the use of situations in NPs with intensional relative clauses, see Moltmann (to appear).
In (42a), the circumstances are not determined by a general condition, such as the one constitutive of John’s needs, but rather they are determined by a particular event of promising (note that John may have promised different books on different occasions). Similarly in (42b), they depend on a particular event of asking. In (42a), the circumstances are those that satisfy a particular promise, and in (42b) the circumstances are those that satisfy a particular demand. A promise is not the same as an act or promising, though, and a demand is not an act of demanding; rather they are the (non-enduring) ‘products’ of acts or promising or demanding in the sense of Twardowski (1999). Products, not acts or states, have satisfaction conditions (Moltmann 2013a, 2014, 2017). This means that the individual concepts denoted by IR-NPs depend on the product of an event argument of the intensional verb in question. They will be functions mapping situations exactly satisfying the product of the event in question to individuals of the relevant sort. For example in (42a), the individual concept would be the function mapping a circumstance exactly satisfying a particular promise to a book John writes in that circumstance. Such truth-making situations cannot be viewed as circumstances of evaluation, though, as standardly conceived.

A general problem for the analysis in (40b) was that it makes excessive use of individual concepts. There is a second option of analysing the book John needs to write, which appears to avoid that problem. On that analysis, the book John needs to write would involve reconstructing the head noun into the lower position inside the relative clause, as in (40c), yielding the analysis in (40d):

\[(40) \quad \text{c. the book } [\text{John needs to write } \{e \{\text{book}\}_N\}] \]
\[(40) \quad \text{d. the function } f [\text{for any world } w \text{ compatible with the satisfaction of John’s needs, write}_{w}(\text{John, } f(w)) \& \text{book}_{w}(f(w))] \]

In (40c), the first trace would be a trace of individual concepts. This analysis obviously would allow the noun book to remain a predicate of individuals.

This analysis raises the very same problem of uniqueness as the first analysis, however. Moreover, just like the first analysis, it is forced to treat all intensional verbs as modal operators quantifying over worlds.

In addition, in its attempt of avoiding type-shifting the analysis cannot go very far. Even though it is plausible that the head noun reconstructs into the lower position, reconstruction of the functional trope noun into a position inside the relative clause is in general impossible: there is no place inside the relative clause for a noun like impact in (3a), repeated below:

\[(43) \quad \text{the impact of the book John needs to write} \]
Impact will have to be interpreted in the upper position. But this means that it will have to denote a function applying to individual concepts.

6 Variable objects

I will propose an account of functional and IR-NPs based on the notion of a variable object and the notion of a variable trope in particular. The notion of a variable object is due to Kit Fine, though the way I will make use of it is not entirely based on Fine’s published work.

Variable objects are entities that fall under Fine’s (1999) more general notion of a variable embodiment (see also Koslicki 2008). The notion of a variable embodiment for Fine is a central notion in metaphysics and accounts for a great variety of ‘ordinary’ objects. Fine himself (p.c.) also meant to apply the notion of a variable embodiment to the semantic values of functional NPs as in (13) as well as NPs with intensional relative clauses such as the book John needs to write. The notion of a variable embodiment allows an account of functional NPs and IR-NPs that treats them as being of type e, while avoiding treating their referents as abstract functions. It hence avoids type-shifting of predicates.

A variable embodiment, according to Fine, is an entity that allows for the replacement of constituting matter or of parts, and more generally that may have different material manifestations in different circumstances. Organisms and artifacts are variable embodiments, but also entities like ‘the water in the river’. Trees and houses allow for a replacement of constituting matter and thus may have different material realizations at different times. They should not be identified with their constituting matter, but rather are entities associated with a function mapping a time to their material realization, that is, they are variable embodiments. ‘The water in the river’ conceived as a variable embodiment will have different realizations as different water quantities at different times (Fine 1999). Variable embodiments differ from ‘rigid embodiments’, entities which do not allow for a replacement of their immediate parts. An example is a token of the word be, which has as its immediate parts a token of b and a token of e, neither of which can be replaced.

Fine’s theory of variable embodiments as formulated in Fine (1999) applies to variable embodiments that may have different manifestations at different times. But the theory is also meant to apply to entities that have different manifestations in different worlds and in fact may lack a manifestation in the actual world (Fine, p.c.). ‘The book John needs to write’ will be such an entity. It is an entity that has manifestations as different objects in various counterfactual circumstances. The
term ‘variable object’, as used in this paper, is meant to apply to entities that have different manifestations as different objects at different times or in different worlds.

Variable objects generally have properties in a derivative way, on the basis of their manifestations. In particular, a variable object exists at a time or in a world just in case it has a manifestation at that time or in that world. Moreover, a variable object shares its location at a time in a world with that of its manifestation at the time in that world provided it has a manifestation at that time in that world. Finally, a variable object ‘inherits’ time- and world-relative properties from its manifestations in the relevant circumstances. Thus, variable objects are subject to the following conditions, which generalize the conditions on variable embodiments of Fine (1999) from times to circumstances (pairs of times and worlds or situations):

(44)  
   a. **Existence**  
   A variable object \( e \) exists in a circumstance \( i \) iff \( e \) has a manifestation in \( i \).

   b. **Location**  
   If a variable object \( e \) exists in a circumstance \( i \), then \( e \)‘s location in \( i \) is that of its manifestation in \( i \).

   c. **Property Inheritance 1**  
   A variable object \( e \) has a (world- or time-relative) property \( P \) in a circumstance \( i \) in which \( e \) exists if \( e \)‘s manifestation in \( i \) has \( P \) in \( i \).

   (44c) accounts for the obligatory presence of a modal with certain IR-NPs, a condition that I will call, following Grosu & Krifka (2007), the *Modal Compatibility Requirement* (MCR). IR-NPs are subject to the MCR if they stand for a variable object that lacks a manifestation in the actual circumstances, for example the book John needs to write as in (2), as can be seen below:

(45)  
   ??? The book John needs to write has a greater impact than the book he has already written.

The modal in (2) permits predicating the predicate of a manifestation in a circumstance other than the actual one.

(44c) does not yet capture all the properties a variable object may have. It only accounts for what I will call its ‘local properties’. In addition to local properties, which are obtained in the way of (44c), variable objects may have ‘global properties’, that is, properties that they may have on the basis of several of their manifestations at different times (for example properties of change, rise, or increase). Variable objects moreover may have properties that are not time- or world-relative. A variable object may have a property in a time- and world-independent way in virtue of all manifestations having that property. This requires a second condition of property inheritance:
(44) d. **Property Inheritance 2**

A variable object has a property \( P \) (circumstance-independently) if all its manifestations have \( P \) in the circumstances in which they exist.

Such a property can nonetheless be attributed to a variable object relative to a time and a world, assuming that having a property circumstance-independently implies having it at all circumstances. Note that there are certain properties, formal properties such a being an object, that by their own nature can borne only circumstance-independently.

The two conditions on property-inheritance in (44c, d) tell when a variable object inherits properties from its manifestations. Given trope theory, this requires corresponding conditions on when a variable object is the bearer of a particular trope in virtue of its manifestations being bearers of corresponding tropes. On a trope-theoretical view, two objects \( o_1 \) and \( o_2 \) sharing a (fully specific) property translates as \( o_1 \) and \( o_2 \) being bearers of two tropes \( t_1 \) and \( t_2 \) that are exactly similar. Within trope theory, Property Inheritance 1 and Property Inheritance 2 can thus be reformulated follows:

(46) a. **Trope ‘Inheritance’ 1**

A variable object \( o \) that exists in a circumstance \( i \) bears a trope \( t \) in \( i \) if \( o \)’s manifestation in \( i \) bears a trope \( t' \) in \( i \) such that \( t' \) is exactly similar to \( t \).\(^6\)

b. **Trope ‘Inheritance’ 2**

A variable object \( o \) bears a trope \( t \) (circumstance-independently) if for any circumstance \( i \) in which \( o \) has a manifestation, \( o \)’s manifestation in \( i \) bears a trope \( t' \) such that \( t' \) is exactly similar to \( t \) in \( i \).

Variable objects are associated with a (partial) function from circumstances to manifestations, but they are not identical to it. That is why variable objects can bear properties of concrete objects, rather than qualifying as abstract. ‘Ordinary’ objects that are variable embodiments do not generally stand in a 1-1 relation to functions from circumstances to manifestations: not any function from circumstances to manifestations corresponds to a variable objects, but rather variable embodiments are driven by conditions such as those on shape and continuity. It is conceivable that two variable embodiments share the same function from circumstances to manifestations. But this is hardly so for variable objects of the sort ‘the book John needs to write’. There is not much more to variable objects of this sort than what

\(^6\) In Moltmann (2013d), I argue that in such cases the very same trope is inherited. This result is a trope with multiple bearers. This is of course a nonstandard view about tropes.
is given by the partial function from circumstances to objects. It is reasonable to assume that there is a unique variable object corresponding to the partial function from circumstances to objects. Variable objects are then ordered according to the ordering among their associated functions: a variable object $o_1$ is part of a variable object $o_2$ iff the function associated with $o_2$ is an extension of the function associated with $o_2$. This relation is important when formulating the semantics of IR-NPs: as definite NPs they generally refer to a minimal variable object satisfying the relative clause.

The variable objects that IR-NPs stand for depend on entities like a ‘need’. A need is not a state of needing and thus not a Davidsonian event argument. Crucially, unlike a Davidsonian event or state, an entity like a need has satisfaction conditions. Thus, in the interpretation of an IR-NP, a function \textit{product} needs to be applied to the Davidsonian event, mapping it onto a related entity that has satisfaction conditions. This will allow the embedded sentence to act as a predicate of the product of the event argument. If controlled clauses involve the self-ascription of a property, as on a standard Lewisian account, then the semantics of attitude verbs with an infinitival clausal complement will be as below:\footnote{For independent motivations of conceiving of that-clause-complements of attitude verbs as predicates of the product of the described event see Moltmann (2014).}

\begin{equation}
\begin{align*}
(47) \quad & a. \text{ For an attitude verb } V \text{ and a (subject-)controlled clause } S \\
& [V \ S] = \lambda e x [V(e, x) & S(\text{product}(e), x)]
\end{align*}
\end{equation}

The infinitival clause will have the syntactic structure in (47b) and will denote the relation in (47b), making use of the standard treatment of controlled clauses as expressing properties of agents:

\begin{equation}
\begin{align*}
(47) \quad & b. \text{ [PRO to write } [e]_\text{DP} [\text{book}]_{\text{NP}}]_{\text{CP}} \\
& c. \lambda e x [\forall s (s \vdash e \to (\text{write}_s(x, d) & \text{book}_s(d)))]
\end{align*}
\end{equation}

For the interpretation of IR-NPs, again the assumption needs to be made that the head noun is interpreted in the lower position inside the relative clause; the lower variable will then stand for a variable object, an object to which the relative clause attributes certain properties in particular circumstances. It will refer to the minimal variable object that is a book and something John writes in any situation satisfying John’s need, as in (47e), which is equivalent to (47f):
(47)  
\[ d. \ [\text{the} \ [e] \ [\text{John needs} \ [\text{to write} \ [\text{[e]_D [book]_{NP}}]_{DP}]]_{CP}]]_{DP} = \]
\[ e. \ \min d \exists e (\text{need}(e, \text{John}) \& \forall s (s \models \text{prod}(e) \rightarrow (\text{write}_s(\text{John}, d) \& \text{book}_s(d)))) = \]
\[ f. \ \min d \exists e (\text{need}(e, \text{John}) \& \forall s (s \models \text{prod}(e) \rightarrow (\text{write}_s(\text{John}, F(d, s)) \& \text{books}(F(d, s))))]) \]

There are constructions that make the product of which a clausal complement is predicated linguistically explicit. Thus, the verb *need* alternates with the light-verb construction *have a need*. In fact, Harves & Kayne (2012) argue that the verb *need* is the result of incorporating the copula *have* and the noun *need*. In any case, given the light-verb construction, the product will be available as part of the compositional semantics of the complex predicate *have+a need*. The construction will thus have the following compositional semantics:

(48)  
\[ \text{the e [John has a need [to write [e book]]]} = \]
\[ \min x \exists e (\text{have}(\text{John}, e) \& \text{need}(e) \& \forall s (s \models e \rightarrow (\text{write}_s(\text{John}, d) \& \text{book}_s(d)))) = \]

Using variable objects has a significant advantage over the individual-concept approach to the compositional semantics of functional NPs and IR-NPs. It allows avoiding a type ambiguity among predicates, definite NPs, and pronouns, and it avoids lifting all those expressions to a higher type.

Let us then turn to trope-referring or apparent trope-referring NPs with IR-NPs, as in (49a) and (49b):

(49)  
\[ a. \ \text{the impact of the number of students} \]
\[ b. \ \text{the increase of the number of students} \]

The functional trope nouns *impact* and *increase* in the upper position apply to a variable object and map it onto a trope that is a local trope based on a single circumstance, as in (49a), or a global trope based on a series of circumstances, as in (49b). The two functional trope nouns denote different functions from variable objects to tropes, as indicated below, where $F$ is the function mapping a variable object $o$ and a circumstance $i$ to the manifestation of $o$ in $i$:

(50)  
\[ a. \ \text{For a variable object } o \text{ and a circumstance } i, \ \text{impact}_i(o) = \text{the trope that has } o \text{ as its bearer and is exactly similar to impact}_i(F(e, i)). \]
\[ b. \ \text{For a variable object } o \text{ and a circumstance } \langle w, t \rangle, \ \text{increase}_i(o) = \text{the trope that has } o \text{ as its bearer and is the instantiation of the property } \lambda e' [F(e', w, t_1) \lessdot F(e', w, t_2) \lessdot \ldots \text{ for subintervals } t_1, t_2, \text{ and } t_3 \text{ of } t \text{ and } t_1 < t_2 < t_3 \ldots ] \]
**Impact** can also serve to describe a variable trope, though, in a sentence such as (4b), repeated below:

(51) The impact of the book John needs to write must exceed the impact of the book he has already written.

The *impact of the book John needs to write* stands for a variable trope that has as its manifestation in a relevant circumstance *i* the impact of the manifestation of the variable object that is ‘the book that John needs to write’, that is, the impact of a particular book John has written in a non-actual circumstance satisfying John’s needs. *The impact of the book John needs to write* in (51) stands for a variable trope, rather than a single trope because of the obligatory presence of the modal. Here the head noun *impact* applies to a variable object and maps it onto a variable trope. The variability of the trope thus is ‘driven by’ the variability of the bearer. A variable trope driven by the variability of its bearer *o* has as its manifestation in a circumstance *i* the trope *t* that has as its bearer the manifestation of *o* in *i*. The noun *impact* in (51) thus denotes the function mapping a variable object onto a variable trope below:

(52) For a variable object *e* and a circumstance *i* ′,

\[ \text{impact}_{t}(e) = \text{the variable trope } o \text{ such that for any circumstance } i \text{ in which } o \text{ has a manifestation } F(e, i), \text{impact}_{t}(F(e, i)) = \text{the manifestation of } o \text{ in } i. \]

A variable trope that has manifestations only in counterfactual circumstances requires a modal in the main clause in order to be attributed local properties in the first place. As a variable trope, ‘the impact of the book John needs to write’ has manifestations only in non-actual circumstances. Ordinary properties can be true of the variable trope in a circumstance *i* only in virtue of being true of a manifestation of the variable trope in *i* provided the variable trope has a manifestation in *i*. If the circumstance *i* is nonfactual, this means that a modal is required in order to shift the context of evaluation of the predicate to a non-actual context, a context in which the variable trope has a manifestation. Again, the Modal Compatibility Requirement follows straightforwardly from Property Inheritance 1.

7 Exemptions from the Modal Compatibility Requirement

Let us turn to the cases in which a modal is not required in the main clause of a sentence with an IR-NP. The Modal Compatibility Requirement does not hold in (53a) and (53b), in contrast to (54a) and (54b):
Intensional Relative Clauses and the Semantics of Variable Objects

(53)  a. The number of people that can fit into the bus exceeds the number of people that can fit into the car.
     b. The length of the vacation John is allowed to take exceeds the length of the vacation Mary is allowed to take.

(54)  a. The impact of the book John needs to write ?? exceeds / ok must exceed / ok might exceed the impact of the book he has already written.
     b. The elegance of the dress the bridesmaid should wear ?? does not exceed / ok should not exceed the elegance of the dress that the bride will wear.

The question, of course, is why are (53a, b) are not subject to the MCR? At first sight, the generalization seems to be that IR-NPs referring to quantitative tropes are not subject to the MCR. But this is not right. The MCR is in place below:

(55)  ?? The number of people John might invite exceeds the number of people Mary might invite.

Yet the distinction between quantitative and qualitative tropes does matter. This is illustrated by the difference between (56a) and (56b) with a one-place evaluative predicate:

(56)  a. The number of papers a student has to write during this program is too high.
     b. The quality of the paper John must write ?? is very high / must be very high.

I propose an explanation of the exemptions from the MCR based on general conditions on when a variable object is the bearer of a particular sort of trope on the basis of its instances bearing particular tropes. The reason why quantitative tropes allow for an exemption from the MCR will be ontological. Quantitative tropes enter relations of exact similarity more easily than qualitative tropes. This is what permits a variable object to act as the bearer of a quantitative trope. Let us take (53a). It is quite plausible that the same number of people fit into the bus / the car in the various relevant circumstances, or at least that this is how agents generally perceive things. This means that the number tropes in the relevant circumstances are exactly similar. Given the condition of Trope Inheritance 2, the variable object itself will bear an exactly similar number trope. Trope Inheritance 2 allows a variable object to be the bearer of a single trope. Generally such a trope has to be a quantitative trope and cannot be a qualitative trope. That is because exact similarity among qualitative tropes is unlikely to obtain, given that natural language predicates in general do not express natural qualitative properties, but unspecific, determinable ones. Thus, in (54a), for example, it can hardly be the case that the
impact of the book John writes in a situation satisfying John’s needs is the very same as the impact of the book he writes in any other situation satisfying his needs. Similarly, in (54b) it will hardly be the case that the elegance of the bridesmaid’s dress in one situation satisfying the relevant conditions is the very same as the elegance of her dress in any other situation satisfying the relevant condition.

This account made use of general ontological conditions on variable objects, Trope Inheritance 2, to explain the exemption from the MCR. This in itself can be considered further support for the ontological account of IR-NPs as such.

8 The Grosu/Krifka construction

Let us finally turn to the construction discussed by Grosu & Krifka (2007) in (3) repeated again below:

(57) The gifted mathematician John claims to be could solve this problem in no time.

Let me call NPs of the type in (57) ‘G/K-NPs’. Characteristically, G/K-NPs involve a copula such as be, become or remain:

(58) a. The mathematician John wants to become should be able to solve the problem in no time.
    b. The honest person that John should remain would pay back his debts in time.

The construction also involves an intensional verb such a claim or a modal verb of necessity or possibility, as also in the examples below:

(59) a. The student John has to become in order to satisfy his parents will have to be very disciplined.
    b. The painter John could have become might have initiated a new art movement.

---

8 This means that be in G/K-NPs is not the be of identity. Further evidence for that is that is identical to is rather unacceptable in that construction:

(i) ??? The mathematician John claims to be identical to would be able to solve the problem in no time.

This constitutes a difficulty for Grosu/Krifka’s analysis for whom be in G/K-NPs expresses the identification of individual concepts (at the relevant circumstances).

9 According to Grosu & Krifka (2007), the construction allows only for modal verbs of necessity. I could not agree with that judgment.
There are significant similarities between the Grosu-Krifka construction and other IR-NPs. Both must involve an intensional verb in the relative clause, both must be definite, and both are subject to the MCR, as illustrated for G/K-NPs below:

(60)  
   a. ??? The student John is in order to satisfy his parents will have to be very disciplined.
   b. ??? A famous painter John could have become might have initiated a new art movement.
   c. ??? The gifted mathematician John claims to be has solved the problem in no time.

Moreover, as Grosu & Krifka (2007) note, their construction allows for anaphora support, just like IR-NPs. Thus, (57) can be continued by (61):

(61) He would have no difficulty with it at all.

As discussed earlier, Grosu & Krifka (2007) analyse G/K-NPs in terms of individual concepts. Roughly, on their analysis, *the gifted mathematician John claims to be* refers to the smallest partial function that maps a world satisfying John’s claim to an entity that is John and a gifted mathematician in that world.

There are the same arguments in favor of conceiving of the referents of G/K-NPs as variable objects rather than individual concepts. In particular, G/K-NPs can also act as the bearers of tropes, in examples such as the following:

(62)  
   a. The talent of the mathematician John claims to be would exceed the talent of the mathematician Mary hopes to become.
   b. The influence of the poet John could have become would by far exceed the influence of the painter John did become.
   c. The honesty of the person John should remain would consist in him paying back his debts.

Cast in terms of variable objects, the subject of (57) refers to the variable object that has a manifestation in any circumstance exactly satisfying John’s claim, namely as an entity that is John and a gifted mathematician.

Thus it is reasonable to carry over the analysis in terms of variable objects to the Grosu/Krifka-construction. There is one particular challenge the construction poses, though, and that concerns the interpretation of the copula. The head noun of the construction should be interpreted in the predicate position of the copula verb. The copula verb then must expresses the identity of the subject referent with the manifestation of the variable object at the relevant circumstance. Of course, this is not compatible with the standard view according to which the predicative complement of the copula verb expresses a property predicated of the subject referent.
There is a recent view, however, concerning copula constructions with indefinite NP complements according to which indefinite complements of the copula do not express an ordinary property to be predicated of the subject referent, but rather define a set so that the subject referent will be identified with its members (Beyssade & Dobrovie-Sorin 2005). This view was meant to explain semantic differences between full indefinite copula complements as in (63a) and bare noun complements as in (63b):

(63) a. John is a mayor.
    b. John is mayor.

This view of the copula with an indefinite NP helps solve the present puzzle, namely if the representation of the full indefinite complement D N′ of a copula verb will not be the simple property λx [N′(x)], but rather the property λy [∃x (N′(x) & y = x)], as below:

(64) For a copula verb V,
    \[ V a N′ = λz [V(z, λy [∃x (N′(x) & y = x)])] \]

The interpretation of the head noun in the lower predicative position can then be considered that of contributing the restriction of the variable that is bound by the relative clause operator. This will give the interpretation of (65a) as in (65b), which is equivalent to (65c), to (65d), and in turn to (65e):

(65) a. the [mathematician [John claims [PRO to be [e mathematician]]]]
    b. min x [∃e (claim(e, John) &
        [PRO to be [e mathematician]][product(e), John)])]
    c. min x [∃e (claim(e, John) &
        ∀s (s ⊨ product(e) → be_s(John, λy [M(x) & x = y)]))]
    d. min x [∃e (claim(e, John) &
        ∀s (s ⊨ product(e) → λy [M(x) & x = y]_s(John)))]
    e. min x [∃e (claim(e, John) &
        ∀s (s ⊨ product(e) → M_s(x) & x = _s John))]

In (65e) the identity symbol is relativized to a circumstance, allowing for the identity of the manifestation of the variable object with John at the circumstance in question.
9 Intentional verbs

There is a different case of an exemption from the MCR, illustrated below:

(66) The originality of the paper John wants to write exceeds the originality of the papers he has so far written.

Here the relative clause contains a verb that has an interpretation not as an intensional verb, but as an intentional verb. This case is an entirely different one from (4a). In (66), the trope ‘the originality of the paper John wants to write’ has as its bearer an intentional object, a nonexistent ‘object of thought’, rather than a variable object (Moltmann 2015). An intentional object is fully present in the world in which the act occurs on which the intentional object depends. It is not an object that has different manifestations in worlds that may exclude the actual one. Only psychological verbs allow for reference to intentional objects, modal verbs of absence like need do not. The latter can involve only variable objects and hence are subject to MCR:

(67) a. The house John needs to live in must be / ??? is huge.
    b. The house John imagines to be living in is huge.

We can also note that transitive intentional and intensional verbs display the very same contrast:

(68) a. The house John is thinking of is huge.
    b. The house John needs must be / ??? is huge.

Obviously, IR-NP transitive intensional verbs involve a similar semantics to that of clausal complement-taking verbs.\(^\text{10}\)

10 See also Moltmann (1997, 2013a) for the semantics of intensional transitive verbs involving variable objects and Moltmann (2015) for the semantics of intentional transitive verbs.

10

10

10 Conclusion

The aim of this paper was to show that the notion of a variable object allows for an account of an otherwise very puzzling construction of apparent trope-referring terms. The notion of a variable object as such is not a peculiar notion, though, invoked only for the analysis of that construction. Rather, it falls under the more general and ontologically central notion of a variable embodiment (in Fine’s metaphysics). As such, it is subject to the very same ontological conditions as drive variable embodiments in general.
References


